Pre-Calculus 11
Chapter 2: Trigonometry

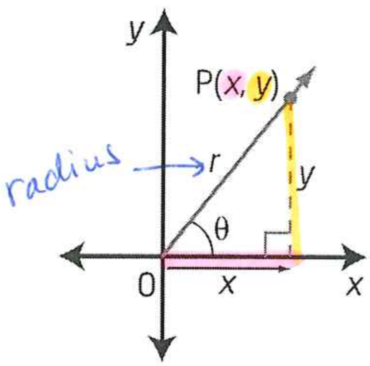
2.2 Trigonometric Ratios of Any Angle (I)

In the past, we have used 3 different trigonometric ratios: sine, cosine, & tangent

In order to remember the ratio represented by each, we use the acronym SOH CAH TOA.

$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

Let's consider θ to represent an angle in standard position, with a point P(x,y) on the terminal arm.



In this case, we can re-write each of the trigonometric ratios in terms of x, y, and r as follows:

$\sin\theta = \frac{y}{r}$ $\qquad \cos\theta = \frac{x}{r}$ $\qquad \tan\theta = \frac{y}{x}$

In each quadrant, different trigonometric ratios are either positive or negative.

$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

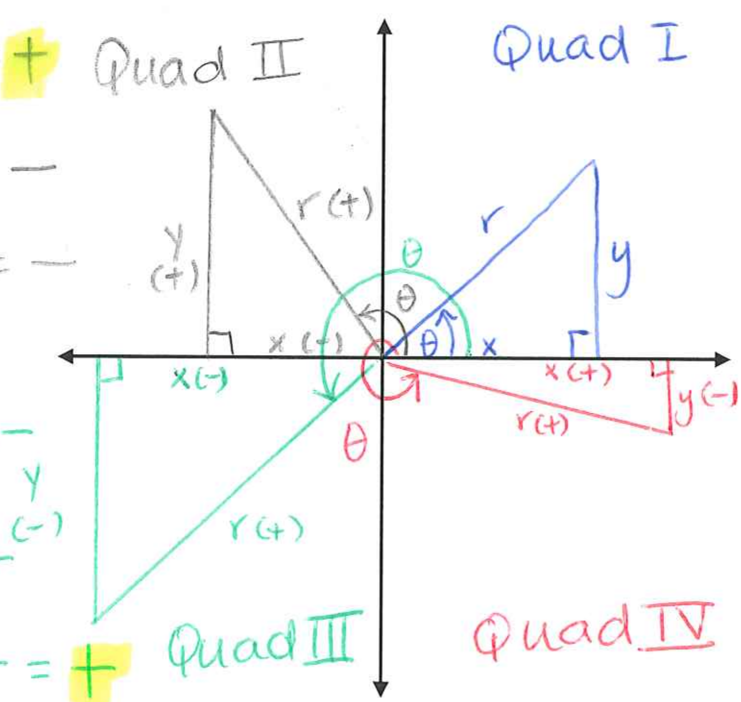
$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

$\tan\theta = \frac{y}{x} = \frac{+}{-} = -$

$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

$\tan\theta = \frac{y}{x} = \frac{+}{+} = +$



$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

$\tan\theta = \frac{y}{x} = \frac{-}{-} = +$

$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

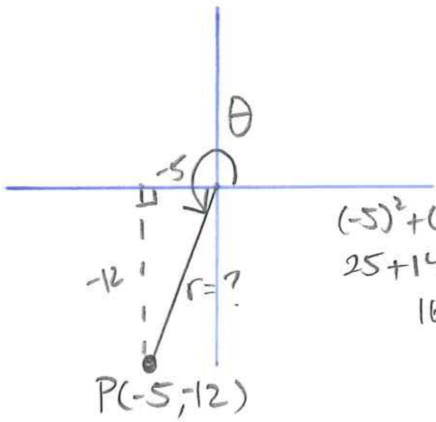
$\tan\theta = \frac{y}{x} = \frac{-}{+} = -$

Which trigonometric ratio is positive in each quadrant?

Example: The point P (-5, -12) lies on the terminal arm of an angle in standard position.

a) Sketch θ in standard position.
b) Determine the exact trigonometric ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$.

a)



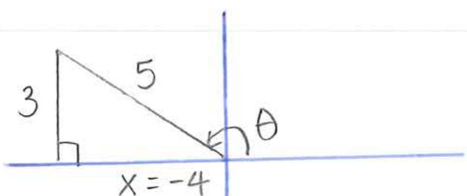
$(-5)^2+(-12)^2=r^2$
$25+144=r^2$
$169=r^2$
$r=\pm 13$
$r=13$ (positive only)

b) $\sin\theta = \frac{y}{r} = \frac{-12}{13}$

$\cos\theta = \frac{x}{r} = \frac{-5}{13}$

$\tan\theta = \frac{-12}{-5} = \frac{12}{5}$

Example: The angle θ is in the second quadrant and $\sin\theta = \frac{3}{5}$. Find the other two primary trigonometric ratios for θ.



$x^2+3^2=5^2$
$x^2=25-9$
$x^2=16$
$x=\pm 4$

$\cos\theta = -\frac{4}{5}$

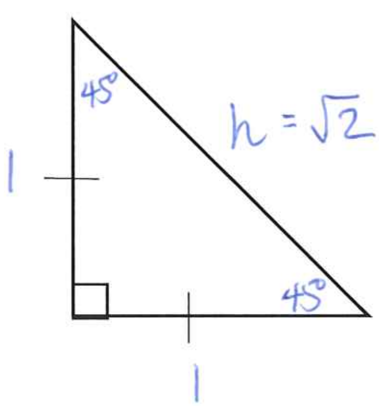
$\tan\theta = -\frac{3}{4}$

Special Triangles:

Next day, we will look at determining trigonometric ratios of some **special angles**.

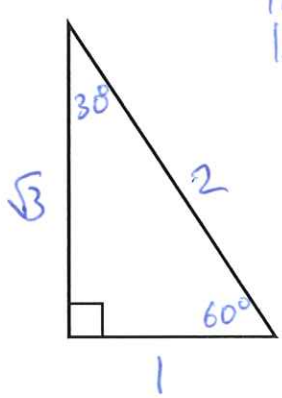
In order to prepare for this, we need to be familiar 2 types of **special triangles**.

$45° - 45° - 90°$

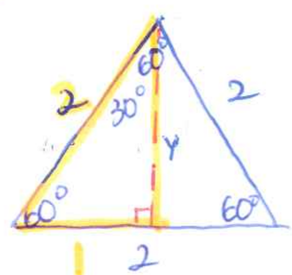


$1^2+1^2=h^2$
$h^2=2$
$h=\pm\sqrt{2}$
$h=\sqrt{2}$ (side length)

$30° - 60° - 90°$



Consider an equilateral triangle with side length = 2.



$y^2+1^2=2^2$
$y^2=3$
$y=\pm\sqrt{3}$
$y=\sqrt{3}$