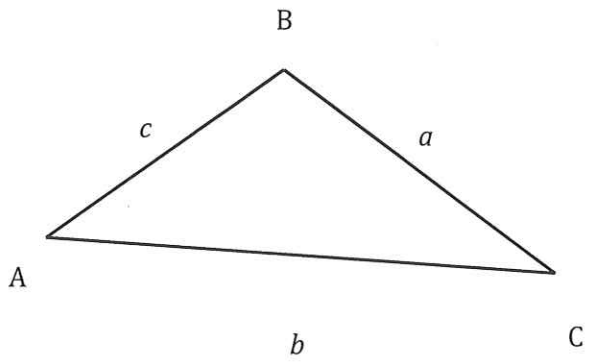
Pre-Calculus 11
Chapter 2: Trigonometry

2.3 The Sine Law (I)

We will now move to a trigonometric application that allows us to solve problems involving triangles that do not contain a 90° angle. Consider the triangle below:

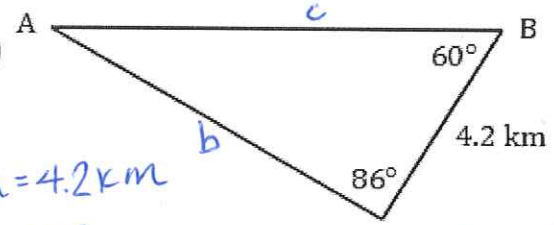


The sine law states that, for all such triangles, it is true that:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \quad \text{OR} \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Example: Determine the unknown side lengths for each triangle (to the nearest tenth).

a)



$a = 4.2\text{km}$

$B = 60°$

$C = 86°$ → $A = 180° - 60° - 86° = 34°$

$\frac{\sin 60°}{b} = \frac{\sin 34°}{4.2}$ · b

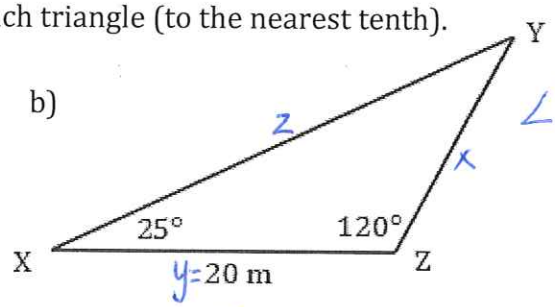
$\frac{4.2 \cdot \sin 34°}{\sin 34°} \cdot b = \frac{\sin 60° \cdot 4.2}{\sin 34°}$

$b = 6.5\text{km}$

$\frac{\sin 86°}{c} = \frac{\sin 34°}{4.2}$

$c = \frac{4.2 \sin 86°}{\sin 34°} = 7.5\text{km}$

b)



$\angle Y = 180° - 120° - 25° = 35°$

$\frac{\sin 35°}{20} = \frac{\sin 25°}{x}$

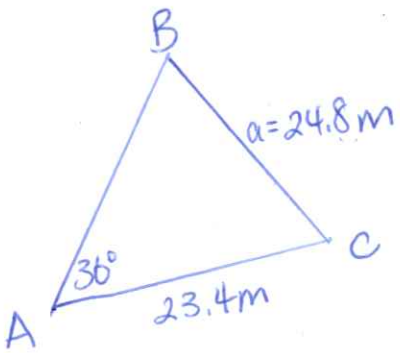
$x = \frac{20 \sin 25°}{\sin 35°} = 14.7\text{m}$

$\frac{\sin 35°}{20} = \frac{\sin 120°}{z}$

$z = \frac{20 \sin 120°}{\sin 35°}$

$z = 30.2\text{m}$

Example: In ΔABC, $\angle A = 36°$, a = 24.8m, and b = 23.4m. Determine $\angle C$ to the nearest degree.



Find $\angle B$ because we know $b = 23.4$m.

$$\frac{\sin B}{23.4} = \frac{\sin 36°}{24.8}$$

$$\sin B = \frac{23.4 \sin 36°}{24.8}$$

$$B = \sin^{-1}\left(\frac{23.4 \sin 36°}{24.8}\right) = 33.68343169°$$

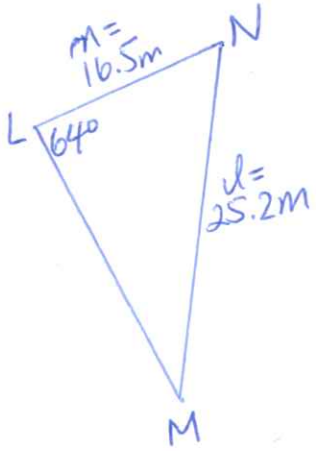
$\angle$s in a Δ add to 180°

$\therefore \angle C = 180° - 36° - \angle B$

$\boxed{\angle C = 110°}$

my diagram is obviously not accurate!

Example: In ΔLMN, $\angle L = 64°$, l = 25.2m, and m = 16.5m. Determine $\angle N$ to the nearest degree.



Find $\angle M$ because we know $m = 16.5$m

$$\frac{\sin M}{16.5} = \frac{\sin 64°}{25.2}$$

$$\sin M = \frac{16.5 \sin 64°}{25.2}$$

$$M = \sin^{-1}\left(\frac{16.5 \sin 64°}{25.2}\right) = 36.05035949$$

$\angle N = 180° - \angle L - \angle M$

$= 79.9496°$

$\boxed{\angle N = 80°}$