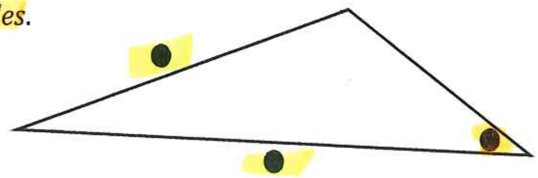
Pre-Calculus 11
Chapter 2: Trigonometry

2.3 The Sine Law (II)

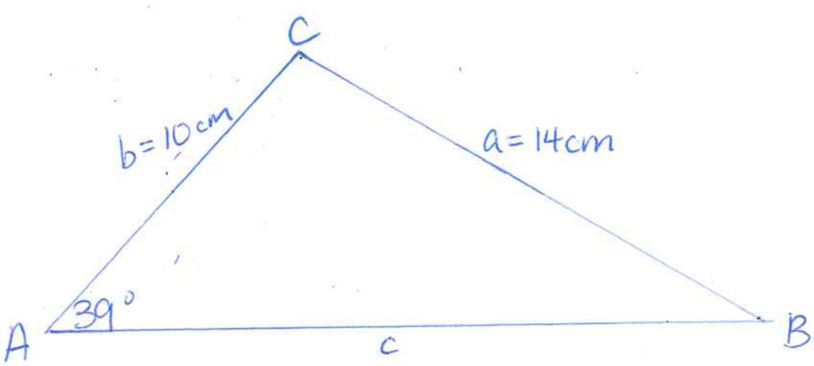
Last day, we saw how we can use the sine law in order to find unknown values in a given triangle.

We now look more closely at the case where we know *the lengths of two sides and the measure of an angle opposite to one of those sides*.



This information can be called **ambiguous** because it is not immediately clear as to which triangle is represented or how many solutions are possible (0, 1, or 2).

Example: In ΔABC, $\angle A = 39°$, $a = 14cm$, and $b = 10cm$. Determine the unknown sides and angles.



$$\frac{\sin 39°}{14} = \frac{\sin B}{10}$$

$$B = \sin^{-1}\left(\frac{10\sin 39°}{14}\right)$$

$$\angle B = 26.71254321°$$

$$\angle C = 180° - \angle B - \angle A = 114.29°$$

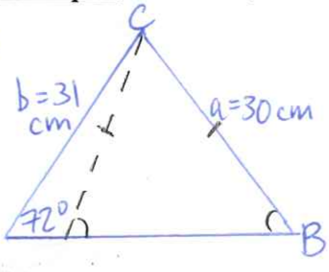
$$\frac{c}{\sin 114.2874568°} = \frac{14}{\sin 39°}$$

$$c = 20.28$$

$\angle B = 27°$, $\angle C = 114°$, $c = 20.3cm$

Example: In ΔABC, $\angle A = 72°$, $a = 30cm$, and $b = 31cm$. Determine the unknown sides and angles.

Case 1:



$$\frac{\sin B}{31} = \frac{\sin 72°}{30}$$

$$B = \sin^{-1}\left(\frac{31\sin 72°}{30}\right)$$

$\angle B = 79.34502618°$

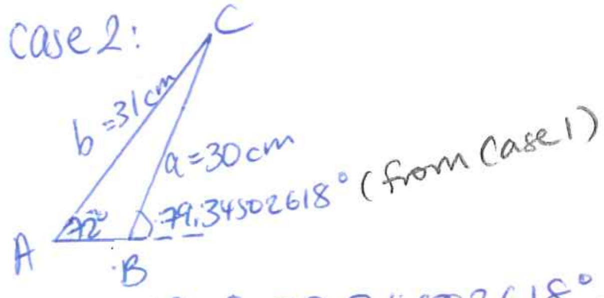
$\angle C = 180° - \angle A - \angle B = 28.65497382°$

$$\frac{c}{\sin C} = \frac{30}{\sin 72°}$$

$$c = \frac{30\sin 28.65497382°}{\sin 72°} = 15.1\,cm$$

$\boxed{\angle B = 79°,\ \angle C = 29°,\ c = 15.1cm}$

Case 2:



$\angle B = 180° - 79.34502618° = 100.6549738°$

$\angle C = 180° - \angle A - \angle B = 7.345°$

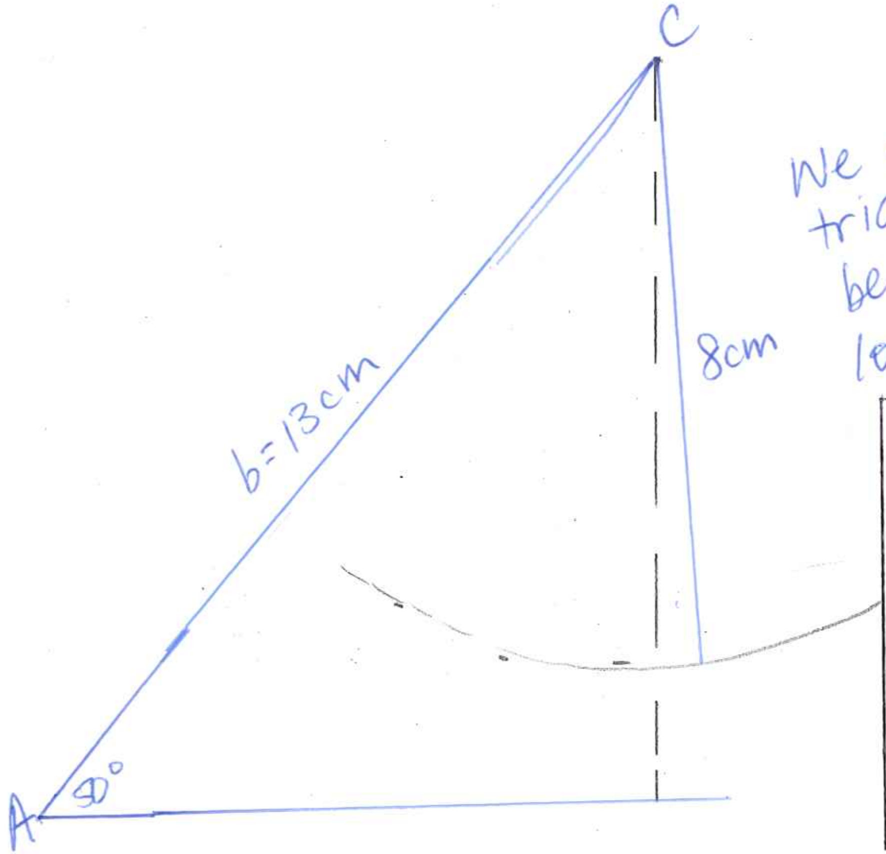
$$\frac{c}{\sin 7.345026°} = \frac{30}{\sin 72°}$$

$c = 4.0327cm$

$$\boxed{\begin{aligned}\angle B &= 101° \\ \angle C &= 7° \\ c &= 4.0\,cm\end{aligned}}$$

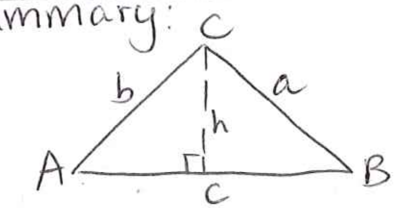
Note: our calculators will only find the acute angle.

Example: In ΔABC, $\angle A = 50°$, $a = 8cm$, and $b = 13cm$. Determine the unknown sides and angles.



We can't make this triangle! a would have to be at least $13\sin 50° = 9.958577461$ cm long to meet the base.

Summary:



1 solution: $a \geq b$

2 solutions: $h < a < b$

0 solutions $a < h$

where $h = b\sin A$