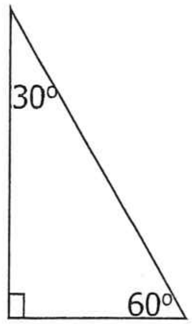
Key

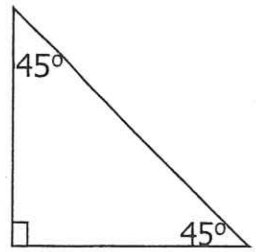
2.8 Relating the Sides of Special Triangles

$30° - 60° - 90°$ Triangle



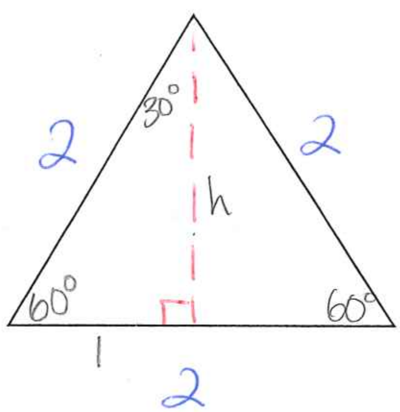
These are the 2 special triangles. The ratios of their sides are what makes them special. There are many applications, especially in science.

$45° - 45° - 90°$ Triangle



Reasoning:

Find the height and area of an equilateral triangle with sides 2 units long.

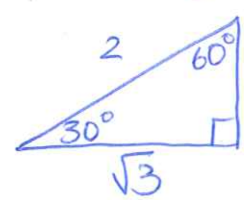


$h^2+1^2=2^2$

$h^2=3$

$h=\sqrt{3}$

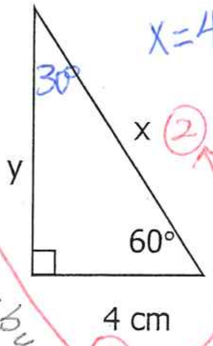
So we get:



If we remember the ratios we can find side lengths very quickly. Note: we can also just use trigonometry.

Example: Find x and y.

a)



$x=4\cdot 2=8cm$

(√3) (2) (1)

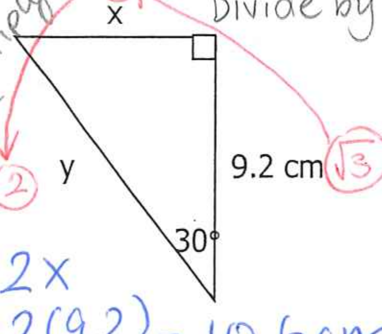
multiply by √3

Think: what do I do to ① to make it ②? multiply by 2!

$y=4\sqrt{3}cm$

↑ called exact form.

b)



(1) (2) (√3)

multiply by 2

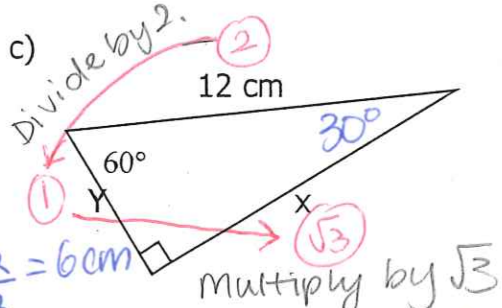
Divide by √3.

$x=\frac{9.2}{\sqrt{3}}=5.3$ cm

$y=2x$

$=\frac{2(9.2)}{\sqrt{3}}=10.6cm$

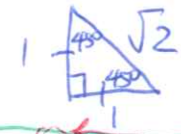
c)



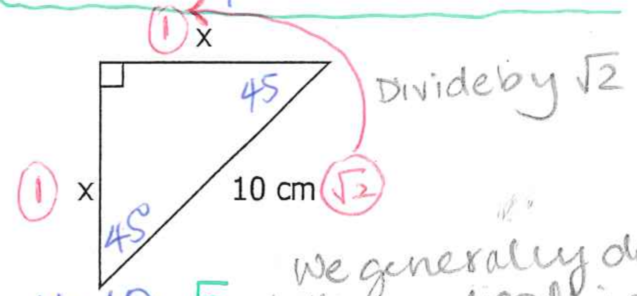
$y = \frac{12}{2} = 6cm$

$x = 6\sqrt{3}$

45°-45°-90° Triangle



d)



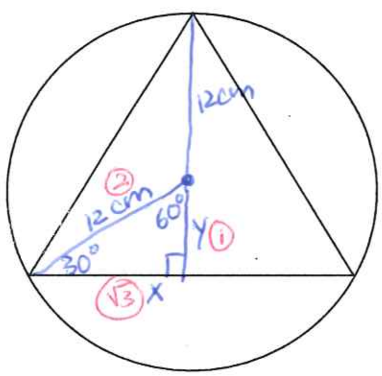
$x = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{10\sqrt{2}}{2}$

$= 5\sqrt{2}$ cm

We generally don't like radicals in the denominator. So we rationalize the denominator.

Example: An Equilateral triangle is inscribed in a circle with radius 12 cm. Find the exact area of the triangle.



$y = \frac{12}{2} = 6cm \rightarrow$ height $= 12 + 6 = 18cm$

$x = 6\sqrt{3} \rightarrow$ base $= 6\sqrt{3} + 6\sqrt{3}$ OR $2(6\sqrt{3})$

$= 12\sqrt{3}$

$\text{Area} = \frac{\text{base} \times \text{height}}{2}$

$= \frac{12\sqrt{3}\,(18)}{2}$

$= 18 \cdot 6\sqrt{3}$

$= 108\sqrt{3}\ cm^2$