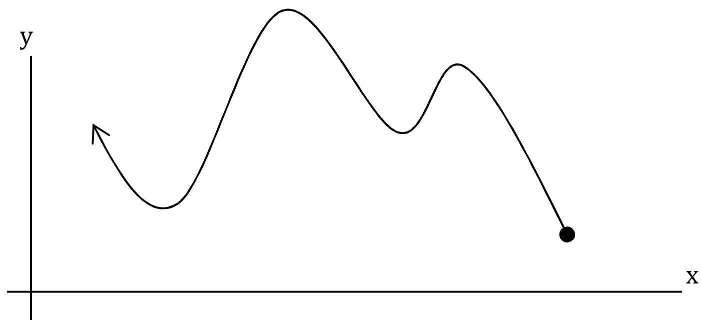
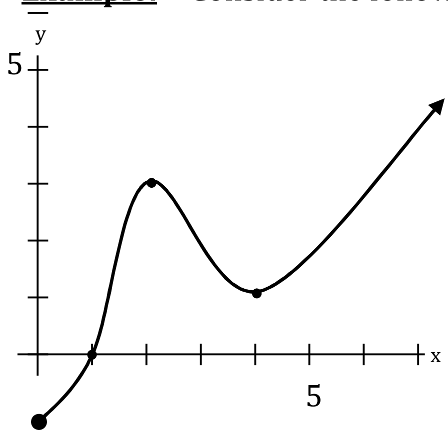
3.1 Maximum and Minimum Values



A function has an ______________________________ on an interval at the point x_0 if ___________________ for all x in the interval. Similarly, a function has an ______________________________ at the point x_0 if _______________ for all x in the interval.

Example: Consider the following function.



Extreme Value Theorem: If a function f is continuous on a closed interval [a,b], then f has both an _____________________________________

_______________________________________on [a,b].

Note: A discontinuous function *could* have maximum and minimum values.

Fermat's Theorem: If f has a local maximum or minimum at x = c, and $f'(c)$ exists, then

____.

Note: We cannot just set $f'(x)=0$ and solve for x to find extreme values. We need to be careful when using Fermat's Theorem. We can use the theorem to *start* looking for extreme values.

A **critical number** of a function f is a number c in the domain of f such that either

____.

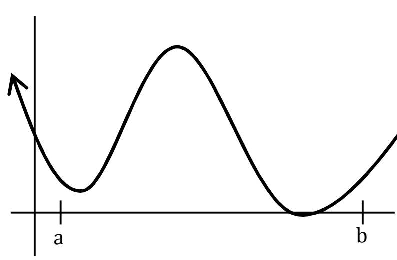
Example: Find the absolute maximum and minimum values of each function on the given interval.

a) $f(x)=\dfrac{x^2-4}{x^2+4}$; $[-4,4]$

b) $g(x)=x^{\frac{1}{3}}-x^{\frac{-2}{3}}$; $[-4,-1]$

3.2 The Mean Value Theorem

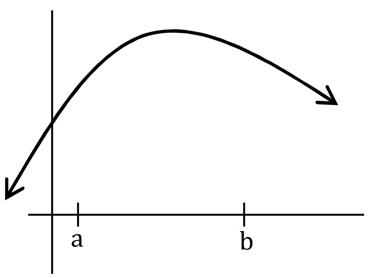
Rolle's Theorem: Let f be differentiable on (a,b) and continuous on [a,b]. If __________, then there is at least one point c, in (a,b) where ___________.



Example: Verify that the hypotheses of Rolle's Theorem are satisfied on the given interval and find all values of c in that interval that satisfy the conclusion of the theorem.

$f(x) = x^2 + 2x + 3; \quad [-3,1]$

Mean Value Theorem: Let f be differentiable on (a,b) and continuous on [a,b], then there is at least one point c in (a,b) where



Example: Verify that the hypotheses of the Mean Value Theorem are satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the theorem. $f(x) = x + \frac{1}{x}; \;\; [3,4]$