Pre-Calculus 11

3.2 Investigating Quadratic Functions in Standard Form

Quadratic functions play a significant mathematical role in describing much of our physical world. The path a projectile (e.g. baseball, cannonball) follows can be modeled using a quadratic function.

A **quadratic function** is any function that can be written in the form: (this form is called standard form)

$y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$

where a, b, c are constants and $a \neq 0$

Example) Which functions are quadratic?

a) $f(x) = (2x-3)(3x+1)$

$= 6x^2 - 7x - 3$

Yes

b) $y = 4x - 2$

No, missing x^2 term.

This is a linear function

c) $f(x) = x(x-3)(x+5)$

$= x(x^2 + 2x - 15)$

$= x^3 + 2x^2 - 15x$

No

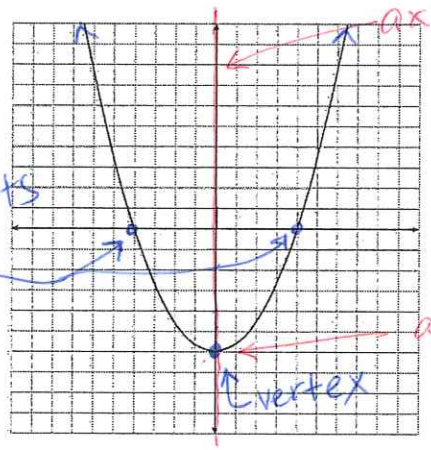
d) $y = 5x^2 + x$

$= 5x^2 + x + 0$

Yes

The graph of a **quadratic function** produces a parabola.

A parabola can open "upwards" or "downwards"



The graph of each quadratic function has properties that make it unique including:

(i) coordinates of the vertex: $(0, -6)$

(ii) equation of the axis of symmetry: $x = 0$

(iii) x-intercept(s): $(-4, 0)$ and $(4, 0)$

(iv) y-intercept: $(0, -6)$

(v) direction of opening: upwards

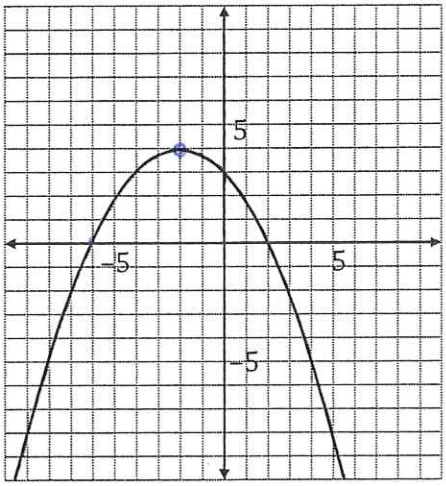
(vi) max/min value: No max, it keeps going up. Minimum value $= -6$

(vii) domain: $\{x \mid x \in \mathbb{R}\}$ all real numbers

(viii) range: $\{y \mid y \geq -6\}$

Example) State the properties of the quadratic functions shown below.

a)



vertex: (-2, 4)

equation of axis of symmetry: $x = -2$

x-intercept(s): (-6,0), (2,0)

y-intercept: (0,3)

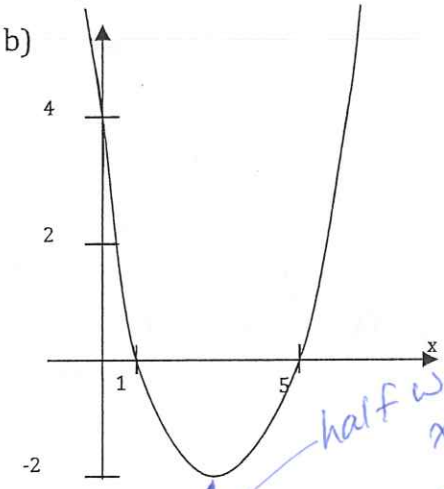
direction of opening: downwards

max/min value: Max = 4

domain: $\{x \mid x \in \mathbb{R}\}$

range: $\{y \mid y \leq 4\}$

b)



halfway between $x=1$ and $x=5$

vertex: (3, -2)

equation of axis of symmetry: $x = 3$

x-intercept(s): (1,0), (5,0)

y-intercept: (0,4)

direction of opening: upwards

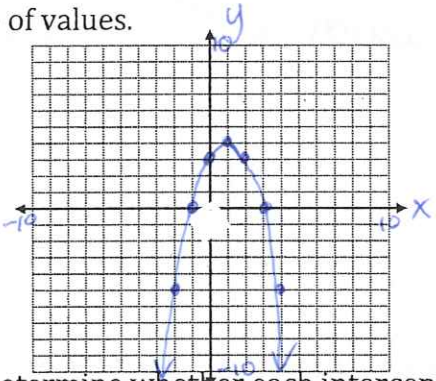
max/min value: Min = -2

domain: $\{x \mid x \in \mathbb{R}\}$

range: $\{y \mid y \geq -2\}$

Example) Sketch the graph of $f(x) = -x^2 + 2x + 3$ using a table of values.

x	$y = f(x)$
-2	-5
-1	0
0	3
1	4
2	3
3	0



Example) How many x-intercepts does each function have? Determine whether each intercept is positive, negative, or zero.

a) A quadratic function with a range of $y \leq -4$.

x-int: none

b) A quadratic function with axis of symmetry $x = 0$ and a minimum value of -3.

x-int: two

1 negative and 1 positive intercept