Pre-Calculus 11

3.3 Completing the Square

Warm Up: Factor each trinomial.

a) $x^2+12x+36$ $6\times6=36$, $6+6=12$
$=(x+6)(x+6)$
$=(x+6)^2$

b) $x^2-14x+49$ $-7\times-7=49$, $-7+-7=-14$
$=(x-7)(x-7)$
$=(x-7)^2$

Recall that we have seen quadratic function in 2 different forms.

A function that is written in the form $y=ax^2+bx+c$ is said to be in **standard form.**

A function that is written in the form $y=a(x-p)^2+q$ is said to be in **vertex form.**

We will sometimes want to change a function from **standard form** to **vertex form.** (Note: It is easier to graph a parabola written in vertex form.)

This can be done using a process called **completing the square**.

Example) Write each quadratic function in vertex form ($y=a(x-p)^2+q$)

a) $f(x)=x^2-6x+9$
$=(x-3)^2$
vertex at $(3,0)$
easy!

b) $y=x^2+8x+2$
cannot factor

To complete the square:

1. Group the first 2 terms.
$y=(x^2+8x)+2$

2. Determine ½ of the x coefficient and square it.
$\left[\frac{1}{2}(8)\right]^2=(4)^2=16$

3. Add and subtract that number inside the brackets.
$y=(x^2+8x+16-16)+2$

4. Remove the last term and combine with the constant.
$y=(x^2+8x+16)+2-16$

5. Factor the expression in brackets.
$y=(x+4)^2-14$ done!

Note: vertex : $(-4,-14)$

c) $y=x^2-8x+5$
$y=(x^2-8x)+5$
$y=(x^2-8x+16-16)+5$
$y=(x^2-8x+16)+5-16$
$y=(x-4)^2-11$

vertex : $(4,-11)$

$\left(\frac{1}{2}(5)\right)^2 = (2.5)^2 = 6.25$

d) $f(x) = x^2 + 5x + 3$

$f(x) = (x^2 + 5x + 6.25) + 3 - 6.25$

$f(x) = (x + 2.5)^2 - 3.25$

$\left[\frac{1}{2}(7)\right]^2 = (3.5)^2 = 12.25$

e) $y = x^2 + 7x - 3$

$y = (x^2 + 7x + 12.25) - 3 - 12.25$

$y = (x + 3.5)^2 - 15.25$

Remember: A vertex can have decimal places

f) $y = 2x^2 - 12x + 11$

1. Group the first 2 terms.

$y = (2x^2 - 12x) + 11$

2. Remove the x^2 coefficient from the first 2 terms.

$y = 2(x^2 - 6x) + 11$

3. Determine ½ of the x coefficient and square it.

$\left(\frac{1}{2}(6)\right)^2 = (3)^2 = 9$

4. Add and subtract that number inside the brackets.

$y = 2(x^2 - 6x + 9 - 9) + 11$

5. Remove the last term and combine with the constant.*

$y = 2(x^2 - 6x + 9) + 11 - 9(2)$

6. Factor the expression in brackets.

$y = 2(x - 3)^2 - 7$

$\left[\frac{1}{2}(4)\right]^2 = (2)^2 = 4$

g) $y = 5x^2 - 20x + 7$

$y = 5(x^2 - 4x) + 7$

$y = 5(x^2 - 4x + 4) + 7 - 4(5)$

$y = 5(x - 2)^2 - 13$

$\left[\frac{1}{2}(-4)\right]^2 = (-2)^2 = 4$

h) $f(x) = -3x^2 + 12x - 7$

$f(x) = -3(x^2 - 4x) - 7$

$f(x) = -3(x^2 - 4x + 4) - 7 - 4(-3)$

$f(x) = -3(x - 2)^2 + 5$

$\left[\frac{1}{2}(10)\right]^2 = (5)^2 = 25$

i) $f(x) = 0.5x^2 + 5x - 4$

$f(x) = 0.5(x^2 + 10x) - 4$

$f(x) = 0.5(x^2 + 10x + 25) - 4 - 25(0.5)$

$= 0.5(x + 5)^2 - 16.5$