Pre-Calculus 11

3.3 Maximum and Minimum Problems

Recall:

1) The maximum/minimum value of a quadratic function is the y-coordinate of the vertex.
2) The vertex of a quadratic function can be determine by completing the square.

Example: Two numbers have a difference of 10. Their product is a minimum. Find the numbers.

Steps: let the 2 numbers be x and y.

1. Write an expression for the maximized or minimized quantity.

$P = xy$

2. Determine a second relationship and solve for one of the variables.

$y - x = 10$

$y = x + 10$

3. Substitute this relationship into the original expression.

$P = x(x+10)$

$P = x^2 + 10x$

4. Complete the square to determine the maximum or minimum value and where it occurs.

$P = (x^2 + 10x + 25) - 25$

$P = (x+5)^2 - 25$

vertex: $(-5, -25)$
opens upward

5. Answer the question.

minimum of -25 when $x = -5$

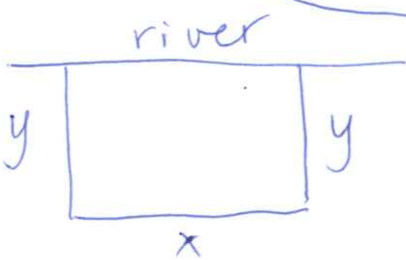
$y = x + 10$

$y = -5 + 10$

$y = 5$

The numbers are -5 and 5.

Example: A rectangular lot is bounded by a river and 80m of fence. Determine the dimensions of the largest possible lot. ← largest area



$A = xy$ and $x + 2y = 80$

$x = -2y + 80$

$A = (-2y + 80)y$

$A = -2y^2 + 80y$

$A = -2(y^2 - 40y)$ — $\left[\frac{1}{2}(40)\right]^2 = (20)^2 = 400$

$A = -2(y^2 - 40y + 400 - 400)$

$A = -2(y-20)^2 - 400(-2)$

$A = -2(y-20)^2 + 800$

opens down
vertex: (20, 800)

Maximum area of 800 when $y = 20$ and

$x = -2y + 80$
$= -2(20) + 80$

$x = 40$.

Dimensions: 20m × 40m

Example: The daily profit of an ice cream stand is given by the function $P = -30x^2 + 120x + 625$ where P is the profit and x is the price of one ice cream cone (in dollars). What should the selling price of an ice cream cone be for maximum daily profit? What is the maximum daily profit?

$P = (-30x^2 + 120x) + 625$

$P = -30(x^2 - 4x) + 625$

$P = -30(x^2 - 4x + 4 - 4) + 625$

$P = -30(x-2)^2 + 625 - 4(-30)$

$P = -30(x-2)^2 + 745$

opens down
vertex (2, 745)

Maximum profit: \$745
price of cone \$2.