3.3 How Derivatives Affect the Shape of the Graph
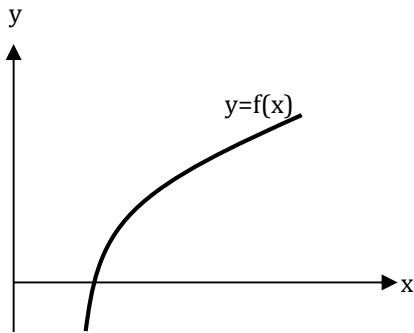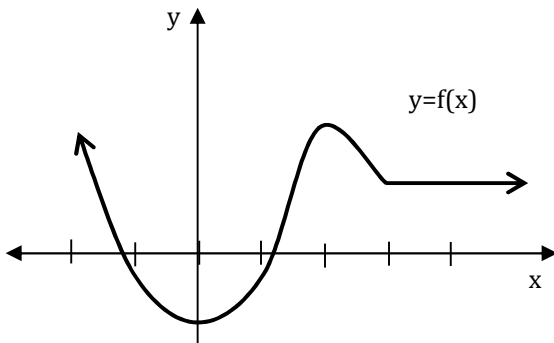

1. Consider $y = f(x)$

2. Consider $f'(x)$

Increasing and Decreasing Functions:

f(x) is increasing:

f(x) is decreasing:

f(x) is constant:

Let f(x) be a function that is continuous on [a,b] and differentiable on (a,b).

a) If $f'(x) > 0$ for $x \in (a,b)$, then $f(x)$ is _____

b) If $f'(x) < 0$ for $x \in (a,b)$, then $f(x)$ is _____

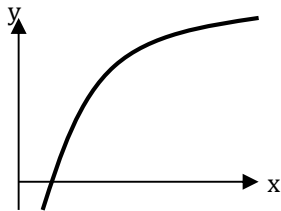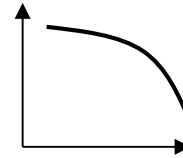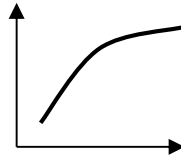c) If $f'(x) = 0$ for $x \in (a,b)$, then $f(x)$ is _____
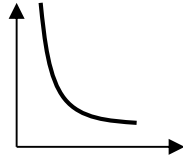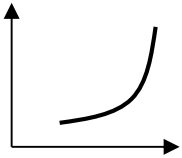
The First Derivative Test:

a) If _____ to the left of x_0 and
    _____ to the right of x_0, then
    f has a _____ at x_0.

b) If _____ to the left of x_0 and
    _____ to the right of x_0, then
    f has a _____ at x_0.

c) If the sign of $f'(x)$ is _____ on both sides of x_0, then
    _____ at x_0.

Concavity:

The values of $f'(x)$ are _____ $\Rightarrow f(x)$_____.

The values of $f'(x)$ are _____ $\Rightarrow f(x)$_____.

Note: If $f''(x) > 0$ on (a,b), then $f'(x)$ is _____ on (a,b), and $f(x)$ is
_____ on (a,b).

If $f''(x) < 0$ on (a,b), then $f'(x)$ is _____ on (a,b), and $f(x)$ is
_____ on (a,b).

Points of Inflection:

Inflection points occur where _____

Example: Consider $f(x) = x^4$.

<u>Example:</u>   Find the intervals on which f :   i) is increasing or decreasing.
                                                        ii) has intervals of concavity.
                                                        iii) has points of inflection.

$$f(x) = -x^3 + 12x + 5$$

<u>Second Derivative Test:</u>

a) If _____ and _____ , then f has a

    _____ at x_0 .

b) If _____ and _____ , then f has a

    _____ at x_0 .

c) If _____ and _____ , then what?

<u>Example:</u>   Find the relative extrema of each function.

a) $f(x) = x^4 + 2x^3$

b) $f(x) = x^{2/3}(6-x)^{1/3}$