

3.6 Polynomials of the Form $ax^2 + bx + c$

Example: To expand $(3x+5)(4x+2)$, make an area with dimensions $3x+5$ and $4x+2$.

$$\begin{array}{c}
 \begin{array}{cc} 4x & +2 \\ \hline 3x & | 12x^2 & 6x \\ +5 & | 9x & 10 \end{array} \\
 = 12x^2 + 15x + 10
 \end{array}$$

This "area model" method is just like using the distributive property and multiplying each term in the first binomial by each term in the second binomial.

Example: Expand and simplify.

a) $(3x-5)(2x+3)$

Use the area model:

$$\begin{array}{c}
 \begin{array}{cc} 3x & -5 \\ \hline 2x & | 6x^2 & -10x \\ +3 & | 9x & -15 \end{array} \\
 = 6x^2 - x - 15
 \end{array}$$

b) $(-5a-8)(7-2a)$

Use the area model:

$$\begin{array}{c}
 \begin{array}{cc} -5a & -8 \\ \hline -2a & | 10a^2 & +16a \\ +7 & | -35a & -56 \end{array} \\
 = 10a^2 - 9a - 56
 \end{array}$$

Use the distributive property:

$$\begin{aligned}
 & (3x-5)(2x+3) \\
 &= 6x^2 + 9x - 10x - 15 \\
 &= 6x^2 - x - 15
 \end{aligned}$$

Use the distributive property:

$$\begin{aligned}
 & (-5a-8)(7-2a) \\
 &= -35a + 10a^2 - 56 + 16a \\
 &= 10a^2 - 9a - 56
 \end{aligned}$$

Example: Factor using the area model.

$$\text{a) } 4x^2 + 20x + 9 \quad \begin{array}{r} 2 \times 18 = 36 \\ 2 + 18 = 20 \end{array}$$

2x	+1
4x ²	2x
18x	9

$$(2x+1)(2x+9)$$

$$\begin{array}{r} 10x^2 = -210 \\ -+- = -11 \end{array}$$

$$\text{b) } 6a^2 - 11a - 35$$

3a	5
6a ²	10a
-21a	-35

$$(3a+5)(2a-7)$$

Example: Factor by decomposition.

$$\text{a) } 4x^2 + 11x + 6 \quad \begin{array}{r} 3 \times 8 = 24 \\ -+- = 11 \end{array}$$

$$\begin{aligned} & 4x^2 + 3x + 8x + 6 \\ &= x(4x+3) + 2(4x+3) \\ &= (4x+3)(x+2) \end{aligned}$$

$$\text{b) } 6x^2 - 7x - 10 \quad \begin{array}{r} -12 \times 5 = -60 \\ -12 + 5 = -7 \end{array}$$

$$\begin{aligned} & 6x^2 - 12x + 5x - 10 \\ &= 6x(x-2) + 5(x-2) \\ &= (x-2)(6x+5) \end{aligned}$$

$$\text{c) } 8x^2 - 18x - 5 \quad \begin{array}{r} -20 \times 2 = -40 \\ -20 + 2 = -18 \end{array}$$

$$\begin{aligned} & 8x^2 - 20x + 2x - 5 \\ &= 4x(2x-5) + 1(2x-5) \\ &= (2x-5)(4x+1) \end{aligned}$$

$$\text{d) } 24x^2 - 20x - 24 \quad \begin{array}{r} -9 \times 4 = -36 \\ -9 + 4 = -5 \end{array}$$

$$\begin{aligned} & 4(6x^2 - 5x - 6) \\ &= 4(6x^2 - 9x + 4x - 6) \\ &= 4(3x(2x-3) + 2(2x-3)) \\ &= 4(2x-3)(3x+2) \end{aligned}$$