3.8 Factoring Special Polynomials

Warm Up: Expand and simply

a) $(x+2)(x-2)$

$= x^2 - 4$

b) $(2x-3)(2x+3)$

$= 4x^2 - 9$

One binomial is the **conjugate** of another binomial if it has the same two terms, but the **opposite sign in between** them. When we multiply two conjugates, the result is a **difference of squares**. To factor a difference of squares, we use the pattern below:

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

Example: Factor.

a) $x^2 - 16$

$= x^2 - 4^2$

$= (x+4)(x-4)$

b) $4x^2 - 25$

$= (2x)^2 - 5^2$

$= (2x+5)(2x-5)$

c) $81x^2 - 100y^2$

$= (9x)^2 - (10y)^2$

$= (9x+10y)(9x-10y)$

d) $4x^2 + 25$

$= (2x)^2 + 5^2$

↑ a sum, we can't factor!

e) $x^4 - 16$

$= (x^2)^2 - 4^2$

$= (x^2+4)(x^2-4)$

$= (x^2+4)(x^2-2^2)$

$= (x^2+4)(x+2)(x-2)$

f) $162a^4 - 2b^4$

$= 2(81a^4 - b^4)$

$= 2((9a^2)^2 - (b^2)^2)$

$= 2(9a^2+b^2)(9a^2-b^2)$

$= 2(9a^2+b^2)((3a)^2-b^2)$

$= 2(9a^2+b^2)(3a+b)(3a-b)$

Review: Expand and simplify.

a) $(x+3)^2$

$= (x+3)(x+3)$

$= x^2+6x+9$

b) $(x-5)^2$

$= (x-5)(x-5)$

$= x^2-10x+25$

c) $(2x+5)^2$

$= (2x+5)(2x+5)$

$= 4x^2+20x+25$

d) $(3x-4y)^2$

$= (3x-4y)(3x-4y)$

$= 9x^2-24xy+16y^2$

The result of squaring a binomial is called a perfect square trinomial. A trinomial square can be factored according to the following patterns:

$$a^2+2ab+b^2=(a+b)(a+b)=(a+b)^2$$
$$a^2-2ab+b^2=(a-b)(a-b)=(a-b)^2$$

Example: Identify and factor only the perfect square trinomials.

a) $x^2+8x+16$

$= (x+4)^2$

b) $x^2-10x+25$

$= (x-5)^2$

c) $x^2+11x+36$

d) $4x^2+20x+25$

$= (2x+5)^2$

e) $36x^2+12x+1$

$= (6x+1)^2$

f) $16x^2-56xy+49y^2$

$= (4x-7y)^2$