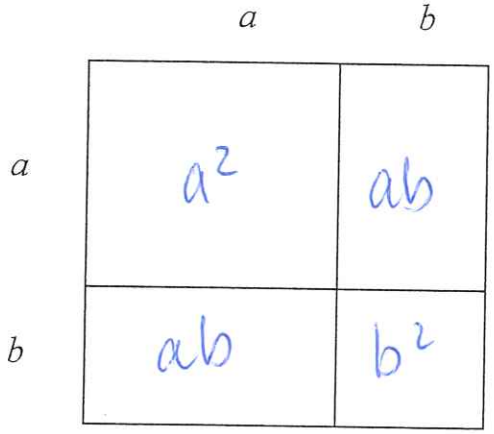
3.8 Factoring Special Polynomials

Objectives:

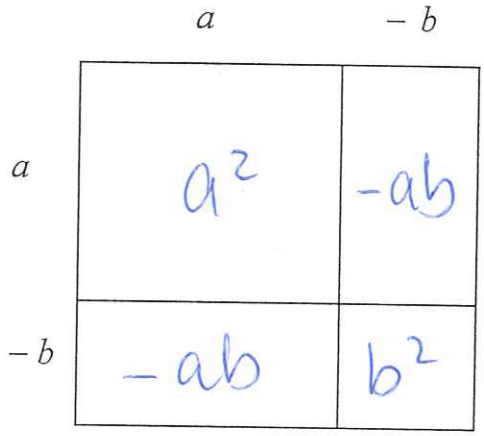
- Investigate some special factoring patterns such as perfect square trinomials and difference of squares.

Recall: $\sqrt{25}=5$, 25 is a perfect square because $5^2=25$. Well, we also have **perfect square trinomials**, such as x^2+2x+1, because $(x+1)^2=x^2+2x+1$.

What is the area of each square with the following side length?



$(a+b)^2 = a^2+2ab+b^2$



$(a-b)^2 = a^2-2ab+b^2$

We can use these patterns to factor **perfect square trinomials**.

Ex. 1 Factor each trinomial.

a) $4x^2+12x+9$

$=(2x+3)^2$

b) $16x^2-8x+1$

$=(4x-1)^2$

c) $4-20x+25x^2$

$25x^2-20x+4$

$=(5x-2)^2$

d) $16-56x+49x^2$

$=(7x-4)^2$

Another example of a special polynomial is a **difference of squares**. A difference of squares is a binomial of the form $a^2 - b^2$.

Think: $a^2 + 0ab - b^2$ $\quad +b \times -b = -b^2$, $+b + -b = 0$

$(a+b)(a-b)$

Ex. 2 Factor each binomial if possible.

a) $x^2 - 25$

$=(x+5)(x-5)$

b) $4y^2 - 9$

$=(2y+3)(2y-3)$

c) $25 - 36m^2$

$=(5+6m)(5-6m)$

d) $81x^2 - 49y^2$

$=(9x+7y)(9x-7y)$

e) $5x^4 - 80y^4$

$=5(x^4 - 16y^4)$

$=5(x^2+4y^2)(x^2-4y^2)$

$=5(x^2+4y^2)(x+2y)(x-2y)$

f) $1 + k^2$ not possible

How do we factor trinomials in two variables that are not *perfect square trinomials* or *difference of squares?*

Ex. 3 Factor each trinomial completely.

a) $2a^2 - 7ab + 3b^2$

$-b \cdot -6b = 6b^2$, $-6 + -6b = -7b$

$=2a^2 - 1ab - 6ab + 3b^2$

$=a(2a-b) - 3b(2a-b)$

$=(2a-b)(a-3b)$

b) $10c^2 - cd - 2d^2$

$-5d \cdot 4d = -20d^2$, $-5d + 4d = -1d$

$=10c^2 - 5cd + 4cd - 2d^2$

$=5c(2c-d) + 2d(2c-d)$

$=(2c-d)(5c+2d)$

c) $8x^2 + 40xy + 50y^2$

$=2(4x^2 + 20xy + 25y^2)$

$=2(2x+5y)(2x+5y)$

	2x	5y
2x	$4x^2$	$10xy$
5y	$10xy$	$25y^2$

d) $x^4 - 13x^2 + 36$ $= (x^2-3)(x^2-4)$

$=(x^2-3)(x+2)(x-2)$

	x^2	-4
x^2	x^4	$-4x^2$
-3	$-9x^2$	36

Practice Work: Pg. 195 #8, 10-13, 15-21