

4.2 Solving Quadratic Equations by Factoring

Warm Up: Factor each expression.

a) $4x^2 - 2x$ Factor GCF
 $= 2x(2x - 1)$

b) $x^2 - 25$ Difference of Squares
 $= (x + 5)(x - 5)$

c) $x^2 + 5x + 6$
 $= (x + 2)(x + 3)$

$$\begin{array}{l} 2 \cdot 3 = 6 \\ 2 + 3 = 5 \end{array}$$

d) $x^2 + x - 6$
 $= (x + 3)(x - 2)$

$$\begin{array}{l} 3 \cdot -2 = -6 \\ 3 + -2 = 1 \end{array}$$

Zero Product Property:

If the product of two real numbers is 0, one or both of the numbers must be 0.

e.g. If $a \cdot b = 0$, then $a = 0$ or $b = 0$ (or both = 0)

To solve a quadratic equation in the form $ax^2 + bx + c = 0$, we may factor the expression and then set either factor equal to zero. The solutions are the roots of the equation.

Example: Determine the roots of each quadratic equation by factoring.

a) $x^2 + 11x + 28 = 0$ $\begin{array}{l} 4 \cdot 7 = 28 \\ 4 + 7 = 11 \end{array}$

$$(x + 4)(x + 7) = 0$$

So $x + 4 = 0$, $x + 7 = 0$
 $x = -4$ $x = -7$

$$\boxed{x = -4, -7}$$

Check: $(-4)^2 + 11(-4) + 28 = 0 \checkmark$
 $(-7)^2 + 11(-7) + 28 = 0 \checkmark$

b) $0 = x^2 + 9x$

$$0 = x(x + 9)$$

So $x = 0$, $x + 9 = 0$
 $x = -9$

$$\boxed{x = -9, 0}$$

Check: $(-9)^2 + 9(-9) = 0 \checkmark$
 $(0)^2 + 9(0) = 0 \checkmark$

c) $x^2 = 81$

$$x^2 - 81 = 0$$

$$(x+9)(x-9) = 0$$

$$x+9=0, x-9=0$$

$$x=-9 \quad x=9$$

$$\boxed{x = -9, 9}$$

check: $(-9)^2 = 81 \checkmark$

$(9)^2 = 81 \checkmark$

d) $x^2 = 2x + 24$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$$\boxed{x = -4, 6}$$

check: $(-4)^2 = 2(-4) + 24$

$$16 = 16 \checkmark$$

and

$$(6)^2 = 2(6) + 24$$

$$36 = 36 \checkmark$$

e) $32 = 4x + x^2$

$$0 = x^2 + 4x - 32$$

$$0 = (x+8)(x-4)$$

$$\boxed{x = -8, 4}$$

f) $3x^2 + 15x + 18 = 0$

$$3(x^2 + 5x + 6) = 0$$

$$3(x+2)(x+3) = 0$$

$$\boxed{x = -3, -2}$$

OR

$$\frac{3x^2 + 15x + 18}{3} = \frac{0}{3}$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$$\boxed{x = -2, 3}$$