Pre-Calculus 12

4.4 The Quadratic Formula (Part 2)

<u>Example:</u> Determine the roots of each equation using the quadratic formula.

a) $x^2 - 6x + 5 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$

$x = \dfrac{6 \pm \sqrt{36 - 20}}{2}$

$x = \dfrac{6 \pm \sqrt{16}}{2}$

$x = \dfrac{6 \pm 4}{2}$

$x = 1, 5$

b) $x^2 - 6x + 9 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$

$x = \dfrac{6 \pm \sqrt{0}}{2}$

$x = 3$

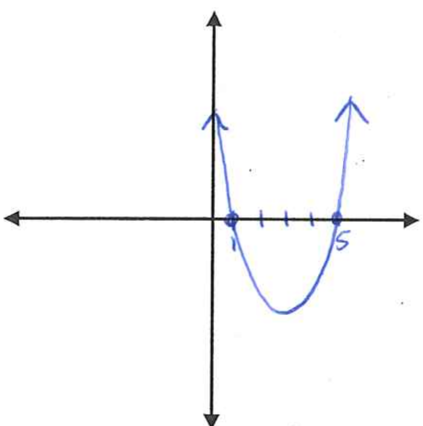
c) $x^2 - 6x + 13 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$

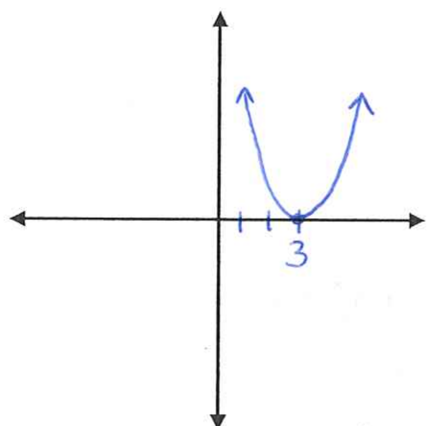
$x = \dfrac{6 \pm \sqrt{36 - 52}}{2}$

$x = \dfrac{6 \pm \sqrt{-16}}{2}$

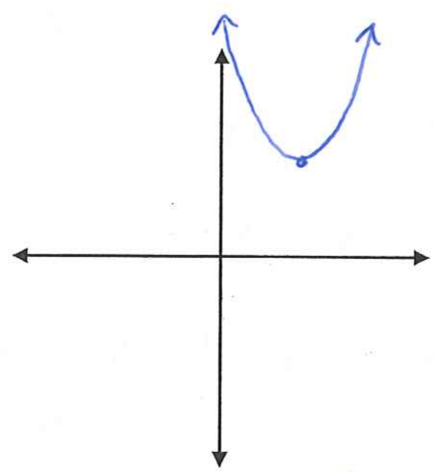
no solution



2 real roots



1 real root



no real roots

As we can see, the value under the radical determines the nature of the roots.

We call the expression $b^2 - 4ac$ the **<u>discriminant</u>** because it discriminates between cases.

If $b^2 - 4ac > 0$, there are 2 real roots

If $b^2 - 4ac = 0$, there is 1 real root

If $b^2 - 4ac < 0$, there are 0 real roots.

$\rightarrow b^2 - 4ac$

Example: Use the discriminant to determine the nature of the roots.

a) $4x^2 - 12x + 9 = 0$

$a = 4$
$b = -12$
$c = 9$

$(-12)^2 - 4(4)(9)$
$= 0$

1 real root

b) $2x^2 + 5x - 1 = 0$

$a = 2$
$b = 5$
$c = -1$

$(5)^2 - 4(2)(-1)$
$= 33 > 0$

2 real roots

c) $x^2 - 2x + 3 = 0$

$a = 1$
$b = -2$
$c = 3$

$(-2)^2 - 4(1)(3)$
$= 4 - 12$
$= -8 < 0$

0 real roots

Example: Determine the roots of the equation using the quadratic formula, to the nearest hundredth.

$0.4x^2 + x + 0.3 = 0$ multiply both sides by 10

$4x^2 + 10x + 3 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-10 \pm \sqrt{100 - 48}}{8}$$

$$x = \frac{-10 \pm \sqrt{52}}{8}$$

$$x = \frac{-10 + \sqrt{52}}{8} \quad \text{and} \quad x = \frac{-10 - \sqrt{52}}{8}$$

$x = -0.35$ $\qquad$ $x = -2.15$