5.1 Working With Radicals (I)

A radical is any expression that can be represented in the form $\sqrt[n]{x}$, where n is called the **index** and x is called the **radicand**.

Note: For radicals with an *even* index, we must ensure that the radicand is not negative.
e.g. For the expression $\sqrt{4-x}$, we know $4-x$ must be greater than or equal to 0.
Thus, $x \leq 4$.

Example: Express each **mixed radical** as an **entire radical**.

a) $5\sqrt{3}$
$= \sqrt{3 \cdot 5 \cdot 5}$
$= \sqrt{75}$

b) $w^3\sqrt{w^2}$
$= \sqrt{w^2 \cdot w^3 \cdot w^3}$
$= \sqrt{w^8}$

c) $2b^2\sqrt{3b}$
$= \sqrt{3b \cdot (2b^2)(2b^2)}$
$= \sqrt{12b^5}$

Example: State any restrictions on the variables for the examples above.
(ie. State what values the variables can have.)

b) $w \in \mathbb{R}$ c) $b \geq 0$

When this process is carried out in reverse, we are said to be simplifying a given radical.
In order for a radical to be considered simplified, both of the following must be true:

1. The radicand contains no factors that can be removed.
2. The radical is not part of the denominator of a fraction.

Example: Express each **entire radical** as a **mixed radical** in simplest form. State any restrictions on the variables.

a) $\sqrt{52}$
$= \sqrt{2 \cdot 26}$
$= \sqrt{2 \cdot 2 \cdot 13}$
$= 2\sqrt{13}$

b) $\sqrt{x^5}$
$= \sqrt{x \cdot x \cdot x \cdot x \cdot x}$
$= x \cdot x\sqrt{x}$
$= x^2\sqrt{x}$

$x \geq 0$

c) $\sqrt{m^4}$
$= \sqrt{m \cdot m \cdot m \cdot m}$
$= m^2$

$m \in \mathbb{R}$

d) $\sqrt{162}$

$= \sqrt{2 \cdot 81}$

$= \sqrt{2 \cdot 9 \cdot 9}$

$= 9\sqrt{2}$

e) $\sqrt[4]{c^7}$

$= \sqrt[4]{\underbrace{c \cdot c \cdot c \cdot c}\cdot c \cdot c \cdot c}$

$= c\sqrt[4]{c^3}$

$c \geq 0$

f) $\sqrt{x^8 y^{11}}$

$= \sqrt{\underbrace{x \cdot x}\cdot\underbrace{x \cdot x}\cdot\underbrace{x \cdot x}\cdot\underbrace{x \cdot x}\cdot \underbrace{y \cdot y}\cdot\underbrace{y y}\underbrace{y y}\underbrace{y y}\underbrace{y y} y}$

$= x^4 y^5 \sqrt{y}$

$x \in \mathbb{R},\ y \geq 0$

g) $\sqrt[3]{54t^7v^{17}}$

$= \sqrt[3]{2 \cdot \underbrace{3 \cdot 3 \cdot 3}\cdot ttttttt\, v \ldots}$

$= 3t^2v^5\sqrt[3]{2tv^2}$

$t \in \mathbb{R},\ v \in \mathbb{R}$

h) $\sqrt{63n^7p^4}$

$= \sqrt{3 \cdot 3 \cdot 7n^7p^4}$

$= 3n^3p^2\sqrt{7n}$

$n \geq 0,\ p \in \mathbb{R}$

<u>Example:</u> Order the radicals $5,\ 3\sqrt{3},\ 2\sqrt{6},\ \sqrt{26}$ from least to greatest without using a calculator.

$5 = \sqrt{25}$ $\quad \sqrt{27}$ $\quad \sqrt{24}$

$\sqrt{24},\ \sqrt{25},\ \sqrt{26},\ \sqrt{27}$

$= 2\sqrt{6},\ 5,\ \sqrt{26},\ 3\sqrt{3}$