Pre-Calculus 11
Chapter 5: Radical Expressions and Equations

## 5.3 Radical Equations

A radical equation is any equation that involves one or more radicals.

There are a few additional requirements when solving radical equations.

When solving a radical equations, we must make sure to:

1. Identify any restrictions on the variable.
2. Identify whether any roots are extraneous.

**Extraneous root**: A solution that does not satisfy the initial restrictions on the variable.

Example: Solve each radial equation. Check for extraneous roots.

a) $(\sqrt{x})^2=(3)^2$ $x\geq 0$

$x=9$

Check: $\sqrt{9}=3$ ✓
OR
$x=9$ is $\geq 0$ ✓

b) $(\sqrt{x})^2=(-5)^2$ $x\geq 0$

$x=25$ ×

Check:
$\sqrt{25}\neq -5$
25 is an extraneous root.
No solution ✓

c) $\sqrt{3x}+2=8$ $x\geq 0$

$(\sqrt{3x})^2=(6)^2$

$3x=36$

$x=12$

check: $\sqrt{3(12)}+2$
$=\sqrt{36}+2$
$=6+2$
$=8$ ✓

d) $(6)^2=(\sqrt{-2x})^2$ $x\leq 0$

$36=-2x$

$x=-18$

Check:

$\sqrt{-2(-18)}$
$=\sqrt{36}$
$=6$ ✓

e) $(2\sqrt{x-1})^2=(x-1)^2$ $x\geq 1$

$4(x-1)=(x-1)(x-1)$
$4x-4=x^2-2x+1$
$0=x^2-6x+5$
$0=(x-1)(x-5)$
$x=1,5$ ✓

Check:

$2\sqrt{1-1}=1-1$
$0=0$ ✓

$2\sqrt{5-1}=5-1$
$2(2)=4$ ✓

f) $-11+2\sqrt{3x-5}=5$ $x \geq 5/3$

$(2\sqrt{3x-5})^2 = 16^2$

$4(3x-5)=256$

$3x-5=64$

$3x=69$

$\boxed{x=23}$ ✓

Check:

$-11+2\sqrt{3(23)-5}=$

$-11+2\sqrt{64}$

$=-11+2(8)$

$=5$ ✓

g) $x-\sqrt{2x+3}=6$ $x \geq -3/2$

$(-\sqrt{2x+3})^2 = (6-x)^2$

$2x+3=(6-x)(6-x)$

$2x+3=36-12x+x^2$

$0=x^2-14x+33$

$0=(x-3)(x-11)$

$\boxed{x=\cancel{3},\ 11}$ ✓

extraneous root

Check:

$3-\sqrt{2(3)+3}$

$=3-\sqrt{9}$

$=0$ X

$11-\sqrt{2(11)+3}$

$=11-\sqrt{25}$

$=6$ ✓

h) $x-\sqrt{5-x}=-7$

$(x+7)^2 = (\sqrt{5-x})^2$

$(x+7)(x+7)=5-x$

$x^2+14x+49=5-x$

$x^2+15x+44=0$

$(x+11)(x+4)=0$

$\boxed{x=\cancel{-11},\ -4}$ ✓

extraneous root

Check:

$-11-\sqrt{5-(-11)}$

$=-11-\sqrt{4}$

$=-13$

X

$-4-\sqrt{5-(-4)}$

$=-4-3$

$=-7$ ✓

*i) $7+\sqrt{3x}=\sqrt{5x+4}+5$

$(2+\sqrt{3x})^2 = (\sqrt{5x+4})^2$

$(2+\sqrt{3x})(2+\sqrt{3x})=5x+4$

$4+4\sqrt{3x}+3x=5x+4$

$(4\sqrt{3x})^2 = (2x)^2$

$16(3x)=4x^2$

$0=4x^2-48x$

$0=4x(x-12)$

$\boxed{x=0,\ 12}$ ✓

Check

$7+\sqrt{0}=\sqrt{0+4}+5$

$7=7$ ✓

$7+\sqrt{3(12)}=\sqrt{5(12)+4}+5$

$13=8+5$ ✓