

6.1 Rational Expressions

We will now look at **rational expressions** and how to carry out operations with them.

Rational Expression: A fraction with a numerator and denominator that are polynomials.

Some examples of rational expressions are: $\frac{1}{x}$, $\frac{x-2}{x^3}$, $\frac{x^2-1}{x^2+2x+1}$, x^2-9 .

When working with rational expressions, it is important to identify any **non-permissible values**.

Non-permissible Values: A value of a variable that makes an expression undefined

- For rational expressions, any value that makes the denominator zero is non-permissible.

Example: Determine the non-permissible values for each rational expression.

a) $\frac{3x}{7yz^2}$ ← we cannot divide by 0! The numerator can be anything

$y \neq 0$
 $z \neq 0$

b) $\frac{3x}{x(3x-2)}$

do not simplify first!

$x \neq 0$
 $3x-2 \neq 0$
 $3x \neq 2$
 $x \neq \frac{2}{3}$

c) $\frac{2x-1}{x^2-x-6}$ ← factor the denominator

$= \frac{2x-1}{(x-3)(x+2)}$

$x \neq -2, 3$

Example: Simplify each rational expression. State any non-permissible values.

Do this BEFORE simplifying!

a) $\frac{12x^3y}{15x^2y^4}$ $x \neq 0, y \neq 0$

$$= \frac{4x}{5y^3}$$

where $x \neq 0, y \neq 0$

b) $\frac{(a+2)(a-7)}{(a-9)(a+2)}$ $a \neq -2, 9$

$$= \frac{a-7}{a-9}, a \neq -2, 9$$

c) $\frac{6-2x}{x^2-9}$ ← difference of squares.

$$= \frac{2(3-x)}{(x+3)(x-3)} \quad x \neq \pm 3$$

$$= \frac{-2(x-3)}{(x+3)(x-3)}$$

$$= \frac{-2}{x+3}, x \neq \pm 3$$

<p>Note: $3-x = -x+3$ $= -(x-3)$ In general: $a-b = -(b-a)$ OR $\frac{(a-b)}{-(b-a)} = -1$</p>

d) $\frac{28-7n}{3n-12}$

$$= \frac{7(4-n)}{3(n-4)} \quad n \neq 4$$

$$= \frac{-7(n-4)}{3(n-4)}$$

$$= -\frac{7}{3}, n \neq 4$$

e) $\frac{2c^2-14c+20}{5c^2-20}$ ← Factor a GCF first

$$= \frac{2(c^2-7c+10)}{5(c^2-4)}$$

$$= \frac{2(c-2)(c-5)}{5(c+2)(c-2)}$$

$$c \neq \pm 2$$

$$= \frac{2(c-5)}{5(c+2)}, c \neq \pm 2$$

f) $\frac{3x-6}{2x^2+x-10}$ ← factor by decomposition

$$= \frac{3(x-2)}{2x^2-4x+5x-10}$$

$$2x^2-4x+5x-10$$

$$= \frac{3(x-2)}{2x(x-2)+5(x-2)}$$

$$2x(x-2)+5(x-2)$$

$$= \frac{3(x-2)}{(x-2)(2x+5)}$$

$$x \neq -\frac{5}{2}, 2$$

$$= \frac{3}{2x+5}, x \neq -\frac{5}{2}, 2$$