6.4 Rational Equations (Part One)

A **rational equation** is an equation containing at least one rational expression.

In order to solve rational equations, we must:

1. Factor denominators.
2. Identify non-permissible values.
3. Multiply both sides by the lowest common denominator (LCD).
4. Isolate the variable and check your solution.

Example: Solve. Identify all non-permissible values

a) $\frac{x-2}{x}+\frac{1}{5}=\frac{-4}{5x}$

$x \neq 0$

LCD $= 5x$ ← the goal is to eliminate the denominators.

$$\frac{5x(x-2)}{x}+\frac{5x\cdot 1}{5}=\frac{5x(-4)}{5x}$$

$$5(x-2)+x=-4$$

$$5x-10+x=-4$$

$$6x=6$$

$$\boxed{x=1}$$

Check: $\frac{1-2}{1}+\frac{1}{5}=\frac{-4}{5}$?

$-1+\frac{1}{5}=-\frac{4}{5}$ ✓

b) $\frac{2}{x^2-4}+\frac{10}{6x+12}=\frac{1}{x-2}$

was $\frac{10}{6}=\frac{5}{3}$

$x \neq \pm 2$

LCD: $3(x+2)(x-2)$

$$\frac{\cdot 2}{(x+2)(x-2)}+\frac{\cdot 5}{3(x+2)}=\frac{1}{(x-2)}$$

$$\frac{3\cancel{(x+2)}\cancel{(x-2)}\cdot 2}{\cancel{(x+2)}\cancel{(x-2)}}+\frac{5\cdot 3\cancel{(x+2)}(x-2)}{3\cancel{(x+2)}}=\frac{1\cdot 3(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$6+5(x-2)=3(x+2)$$

$$6+5x-10=3x+6$$

$$2x=10$$

$$\boxed{x=5}$$

check:

$\frac{2}{5^2-4}+\frac{10}{6(5)+12}=\frac{1}{3}$?

$\frac{2}{21}+\frac{10}{42}=\frac{1}{3}$ ✓

c) $\frac{9}{x-3}-\frac{4}{x-6}=\frac{18}{x^2-9x+18}$

$$\frac{9}{x-3}-\frac{4}{x-6}=\frac{18}{(x-3)(x-6)}$$

$x \neq 3, 6$

LCD: $(x-3)(x-6)$

$$\frac{9\cancel{(x-3)}(x-6)}{\cancel{(x-3)}}-\frac{4(x-3)\cancel{(x-6)}}{\cancel{(x-6)}}=\frac{18\cancel{(x-3)}\cancel{(x-6)}}{\cancel{(x-3)}\cancel{(x-6)}}$$

$$9(x-6)-4(x-3)=18$$

$$9x-54-4x+12=18$$

$$5x=60$$

$$\boxed{x=12}$$

Check:

$$\frac{9}{12-3}-\frac{4}{12-6}=\frac{18}{(12)^2-9(12)+18} \quad ?$$

$$1-\frac{2}{3}=\frac{18}{54}$$

$$\frac{1}{3}=\frac{1}{3} \checkmark$$