Pre-Calculus 11
Chapter 6: Rational Expressions and Equations

6.4 Rational Equations (Part Two)

Example: The sum of two numbers is 16. The sum of their reciprocals is 1/3. Find the numbers.

let the numbers be n and $16-n$ (Note: $n+(16-n)=16$)

$3n(16-n)\frac{1}{n} + \frac{1\cdot 3n(16-n)}{16-n} = \frac{1\cdot 3n(16-n)}{3}$ $n \neq 16$ LCD $3n(16-n)$

$3(16-n)+3n = n(16-n)$

$48-3n+3n = 16n-n^2$

$n^2-16n+48=0$

$(n-4)(n-12)=0$

$n = 4, 12$

The numbers are 4 and 12

Example: It takes 4 hours for Michelle to paint a room. Her younger brother David takes 6 hours to paint the same room. How long would it take them to paint the room working together?

let t = time it takes working together

	Time to paint room	Fraction of work done in 1 hour	Fraction of work done in t hours
Michelle	4 hours	$\frac{1}{4}$	$\frac{t}{4}$
David	6 hours	$\frac{1}{6}$	$\frac{t}{6}$
Together	t	$\frac{1}{t}$	1 ← the whole job

$\frac{t}{4}+\frac{t}{6}=1$ LCD = 12

$\frac{12t}{4}+\frac{12t}{6}=12$

$3t+2t=12$

$5t=12$

$t=\frac{12}{5}$ hours $= 2\frac{2}{5}$ hours $= 2.4$ hours

Example: A train has a scheduled run of 160 km between two cities in Saskatchewan. If the average speed is decreased by 16 km/h, the run will take ½ hour longer. What is the average speed of the train?

let x be the average (current speed)

	Distance (km)	Speed (km/h)	Time (h)
At current speed	160	x	$\frac{160}{x}$
At slower speed	160	$x-16$	$\frac{160}{x-16}$

at slower speed it takes 0.5 h longer.

$$\frac{160}{x-16} = \frac{160}{x} + \frac{1}{2} \qquad x \neq 16$$

LCD: $2x(x-16)$

$$2x(x-16)\cdot\frac{160}{(x-16)} = 2x(x-16)\cdot\frac{160}{x} + 2x(x-16)\cdot\frac{1}{2}$$

$$320x = 320x - 5120 + x^2 - 16x$$

$$0 = x^2 - 16x - 5120$$

$$0 = (x-80)(x+64)$$

$$x = -64, 80$$

↑ it doesn't make sense for average initial speed to be negative.

$x = \boxed{80 \text{ km/h}}$