Pre-Calculus 11
Chapter 7: Absolute Value and Reciprocal Functions

7.2 Absolute Value Functions (Part 1)

Absolute value function: A function that involves the absolute value of a variable.

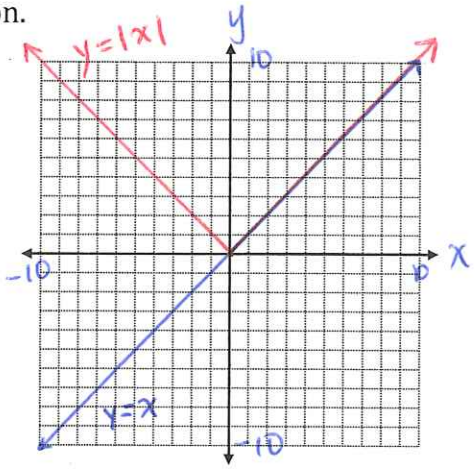
Example: Complete each table of values to sketch each function.

a) $y = x$

x	y = x
−3	−3
−2	−2
−1	−1
0	0
1	1
2	2
3	3

b) $y = |x|$

x	$y = \|x\|$
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3



Determine the domain, range, x-intercepts and y-intercepts of each function.

a) $y = x$

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \in \mathbb{R}\}$
x-int: (0,0)
y-int: (0,0)

b) $y = |x|$

D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$
x-int: (0,0)
y-int: (0,0)

Creating a table of values can be very time consuming. Notice that the points below the x-axis on $y = x$ are reflected above the x-axis on $y = |x|$.

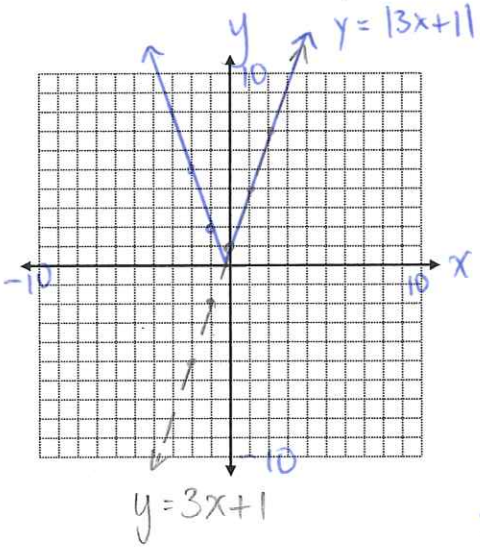
The points on the x-axis or above the x-axis remain the same on both graphs. These are called **invariant points**.

The graph of $y = |x|$ can be defined by two separate pieces.

1. For $x \geq 0$, the graph is $y = x$.
2. For $x < 0$, the graph is $y = -x$

$\therefore$ The **piecewise** definition of $y = |x|$ is $y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Example: Sketch the graph of $y = |3x + 1|$. State the intercepts and the domain and range. Express the function as a piecewise function.



① Graph $y = 3x + 1$

② Reflect anything that falls below the x-axis "up"

$D: \{x \mid x \in \mathbb{R}\}$

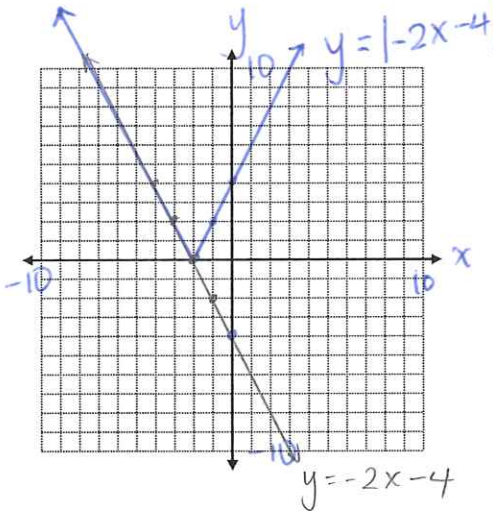
$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

$$y = \begin{cases} 3x+1, & x \geq -\frac{1}{3} \\ -3x-1, & x < \frac{-1}{3} \end{cases}$$

x-int:
let $y = 0$
$0 = 3x + 1$
$x = -1/3$
$(-1/3, 0)$

y-int:
let $x = 0$
$y = 1$
$(0, 1)$

Example: Sketch the graph of $y = |-2x - 4|$. State the intercepts and the domain and range. Express the function as a piecewise function.



① $y = -2x - 4$

$D: \{x \mid x \in \mathbb{R}\}$

$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

x-int

$0 = |-2x - 4|$

$-2x - 4 = 0$

$-2x = 4$

$x = -2$

$(-2, 0)$

y-int:

$y = |-2(0) - 4|$

$y = |-4|$

$y = 4$

$(0, 4)$

$$y = \begin{cases} 2x+4, & x \geq -2 \\ -2x-4, & x < -2 \end{cases}$$