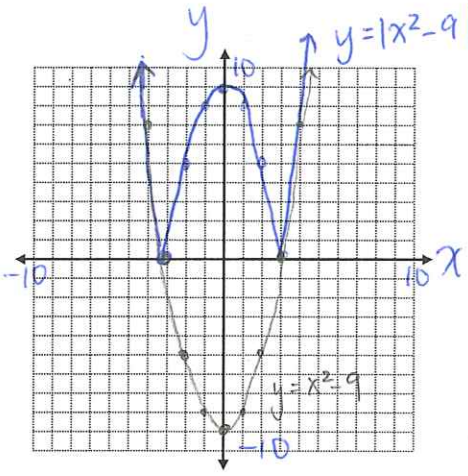
Pre-Calculus 11
Chapter 7: Absolute Value and Reciprocal Functions

7.2 Absolute Value Functions (Part 2)

Recall: $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$

Example: Sketch the graph of $y = |x^2 - 9|$. State the intercepts and the domain and range. Express the function as a piecewise function.



① Sketch $y = x^2 - 9$ (Recall: parabola with a vertex at $(0,9)$)

② Reflect everything that falls below the x-axis 'up'

x-int:
$0 = |x^2 - 9|$
$0 = x^2 - 9$
$x^2 = 9$
$x = \pm 3$
$(3,0)$ and $(-3,0)$

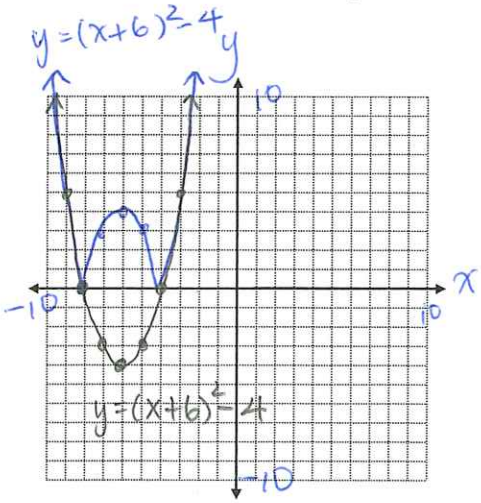
y-int:
$y = |0^2 - 9|$
$y = |-9|$
$y = 9$
$(0,9)$

$D: \{x \mid x \in \mathbb{R}\}$

$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

$$y = \begin{cases} x^2 - 9, & x \leq -3 \text{ or } x \geq 3 \\ -(x^2 - 9), & -3 < x < 3 \end{cases}$$

Example: Sketch the graph of $y = |(x+6)^2 - 4|$. State the intercepts and the domain and range. Express the function as a piecewise function.



① parabola w/ vertex @ $(-6,-4)$

x-int:
$0 = |(x+6)^2 - 4|$
$0 = (x+6)^2 - 4$
$0 = x^2 + 12x + 36 - 4$
$0 = x^2 + 12x + 32$
$0 = (x+4)(x+8)$
$x = -4, -8$
$(-4,0)$ and $(-8,0)$

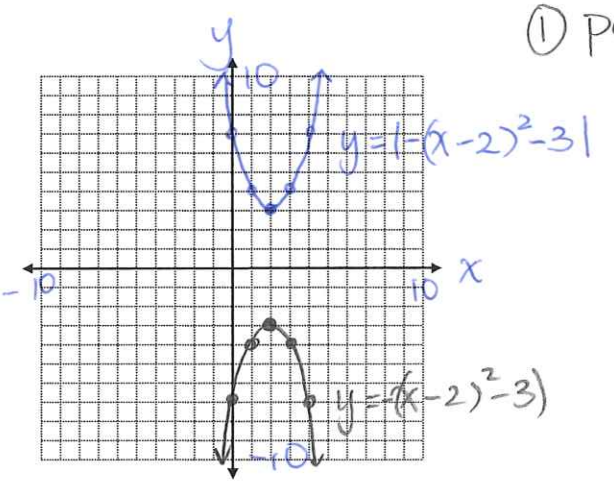
y-int:
$y = |(0+6)^2 - 4|$
$y = |36 - 4|$
$y = 32$ $(0,32)$

$D: \{x \mid x \in \mathbb{R}\}$

$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

$$y = \begin{cases} (x+6)^2 - 4, & x \leq -8 \text{ or } x \geq -4 \\ -[(x+6)^2 - 4], & -8 < x < -4 \end{cases}$$

Example: Sketch the graph of $y = \left|-(x-2)^2 - 3\right|$. State the intercepts and the domain and range. Express the function as a piecewise function.



① parabola: vertex: (2, -3)
opens "down"

x-int:

$0 = |-(x-2)^2 - 3|$

$0 = -(x-2)^2 - 3$

$0 = -(x^2 - 4x + 4) - 3$

$0 = -x^2 + 4x - 4 - 3$

$x^2 - 4x + 7 = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

no solution

(no x-intercepts!)

y-int:

$y = |-(0-2)^2 - 3|$

$y = |-4-3|$

$y = |-7|$

$y = 7$ (0, 7)

$y = -[-(x-2)^2 - 3]$

$\boxed{y = (x-2)^2 + 3}$

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 7, y \in \mathbb{R}\}$