Pre-Calculus 11
Chapter 7: Absolute Value and Reciprocal Functions

## 7.3 Absolute Value Equations

Recall: To **solve** an equation means to find the value of the unknown.

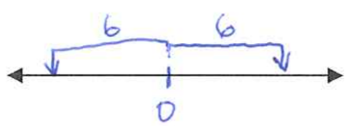
### Method 1: Using the Distance Definition

$|x| \rightarrow$ the distance from x to 0 on a number line

$|x-a| \rightarrow$ the distance from x to a on a number line

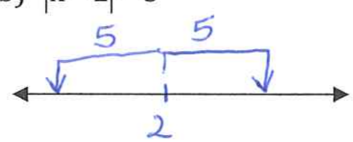
Example: Solve using a number line.

a) $|x|=6$



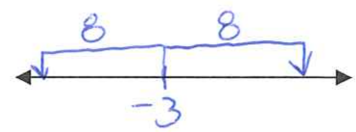

$x=-6,6$

b) $|x-2|=5$




$x=-3,7$

c) $|x+3|=8$




$x=-11,5$

### Method 2: Using Two Cases

When solving absolute value equations, we must consider 2 possible situations:

*Case 1*: The expression inside the absolute value is **positive**.

*Case 2:* The expression inside the absolute value is **negative**.

Example: Solve algebraically.

a) $|x-3|=5$

① $x-3=5$

$\boxed{x=8}$

② $x-3=-5$

$\boxed{x=-2}$

Check: $|8-3|=5$ ✓

$|-2-3|=5$ ✓

$\boxed{x=-2,8}$

b) $|x+5|=4x-1$

① $x+5=4x-1$

$5=3x-1$

$6=3x$

$\boxed{x=2}$

Check:

$|2+5|=8-1$ ✓

② $x+5=-(4x-1)$

$x+5=-4x+1$

$5x=-4$

$\cancel{\boxed{x=-\frac{4}{5}}}$ ← extraneous root

$|-\frac{4}{5}+5|=4(-\frac{4}{5})-1$

$\frac{21}{5} \neq -\frac{16}{5}-1$

$\boxed{x=2}$

$|x-4|+12=9$

$|x-4|=-3$

no solution.

The absolute value of an expression should never be negative.

d) $|x-5|=x^2-8x+15$

① $x-5=x^2-8x+15$

$0=x^2-9x+20$

$0=(x-4)(x-5)$

$\boxed{x=\cancel{4},5}$

② $x-5=-(x^2-8x+15)$

$x-5=-x^2+8x-15$

$x^2-7x+10=0$

$(x-2)(x-5)=0$

$\boxed{x=2},5$

Check: $|4-5|=(4)^2-8(4)+15$ X

x = 4 is extraneous

$|5-5|=(5)^2-8(5)+15$ ✓

$|2-5|=(2)^2-8(2)+15$ ✓

$\boxed{x=2,5}$

e) $|x^2-3x|=2$

① $x^2-3x=2$

$x^2-3x-2=0$

$x=\dfrac{3\pm\sqrt{(-3)^2-4(1)(-2)}}{2(1)}$

$\boxed{x \doteq 3.56,-0.56}$

② $x^2-3x=-2$

$x^2-3x+2=0$

$(x-1)(x-2)=0$

$\boxed{x=1,2}$

Check: $|3.56^2-3(3.56)|=2$ ✓

$|(-0.56)^2-3(-0.56)|=2$ ✓

$|1^2-3|=2$ ✓

$|2^2-3(2)|=2$ ✓

$\boxed{x=1,2,3.56,-0.56}$