Foundations and Pre-Calculus Hon.

7.4 Solving Linear Systems by Substitution

The substitution method:

Step 1: Isolate one variable in one of the equations.

Step 2: Substitute the equation you get from step 1 into the other equation, thus eliminating one of the variables.

Step 3: Solve for one variable using the equation from step 2.

Step 4: Substitute the solution from step 3 into your equation from step 1 to solve for the second variable.

Step 5: Write your solution as an ordered pair (x, y).

Example: Solve each of the following systems of equations using the substitution method.

a)
$$3x + y = 13$$
$$x + y = 3$$

b)
$$5x + 6y = -11$$
$$3x + y = -4$$

1) $y = 13 - 3x$

2) $x + (13 - 3x) = 3$

3) $13 - 2x = 3$
$$-2x = -10$$
$$x = 5$$

4) $y = 13 - 3x$
$$y = 13 - 3(5)$$
$$y = -2$$

5) $(5, -2)$

1) $y = -3x - 4$

2) $5x + 6(-3x - 4) = -11$

3) $5x - 18x - 24 = -11$
$$-13x = 13$$
$$x = -1$$

4) $y = -3x - 4$
$$y = -3(-1) - 4$$
$$y = 3 - 4$$
$$y = -1$$

5) $(-1, -1)$

$$3x + y = 18$$

c) $x + 2y = 11$

1) $y = 18 - 3x$

2) $x + 2(18 - 3x) = 11$

3) $x + 36 - 6x = 11$
$$-5x = -25$$
$$x = 5$$

4) $y = 18 - 3(5) = 3$ 5) $(5, 3)$

d) $-x + 3y = -8$
$2x + 3y = 5$

1) $x = 3y + 8$

2) $2(3y + 8) + 3y = 5$

3) $6y + 16 + 3y = 5$
$$9y = -11$$
$$y = -11/9$$

4) $x = 3(-11/9) + 8$
$$x = 13/3$$

5) $\left(\frac{13}{3}, \frac{-11}{9}\right)$

Note: Multiplying or dividing equations in a linear system by a non-zero number does not change the graphs. So, their point of intersection (ie. the solution) of the linear system remains unchanged.

e) $6\left(\dfrac{1}{2}x + \dfrac{2}{3}y = -1\right)6$

$12\left(y = \dfrac{1}{4}x - \dfrac{5}{3}\right)12$

$$3x + 4y = -6$$
$$12y = 3x - 20$$

$$4y = -3x - 6$$
$$y = \dfrac{-3x}{4} - \dfrac{3}{2}$$

$$12\left(-\dfrac{3}{4}x - \dfrac{3}{2}\right) = 3x - 20$$

$$-9x - 18 = 3x - 20$$

$$2 = 12x$$

$$x = \dfrac{1}{6}$$

$$y = -\dfrac{3}{4}\left(\dfrac{1}{6}\right) - \dfrac{3}{2}$$

$$= -\dfrac{1}{8} - \dfrac{3 \cdot 4}{2 \cdot 4}$$

$$= \dfrac{-13}{8}$$ $\left(\dfrac{1}{6}, \dfrac{-13}{8}\right)$

f) $12\left(\dfrac{7}{4}x + \dfrac{4}{3}y = 3\right)12$

$6\left(\dfrac{1}{2}x - \dfrac{5}{6}y = 2\right)6$

$$21x + 16y = 36$$
$$3x - 5y = 12$$

$$3x = 5y + 12$$
$$x = \dfrac{5}{3}y + 4$$

$$21\left(\dfrac{5}{3}y + 4\right) + 16y = 36$$

$$35y + 84 + 16y = 36$$

$$51y = -48$$
$$y = \dfrac{-48}{51}$$

$$x = \dfrac{5}{3}\left(-\dfrac{48}{51}\right) + 4$$

$$= \dfrac{-240}{153} + 4$$

$$= \dfrac{-80}{51} + \dfrac{204}{51}$$ $\left(\dfrac{124}{51}, \dfrac{-16}{17}\right)$

$$= 124/51$$