Pre-Calculus 11
Chapter 7: Absolute Value and Reciprocal Functions

7.4 Reciprocal Functions (Part Two: Quadratic)

Recall that the reciprocal of a function $y=f(x)$ is $y=\frac{1}{f(x)}$, $f(x)\neq 0$.

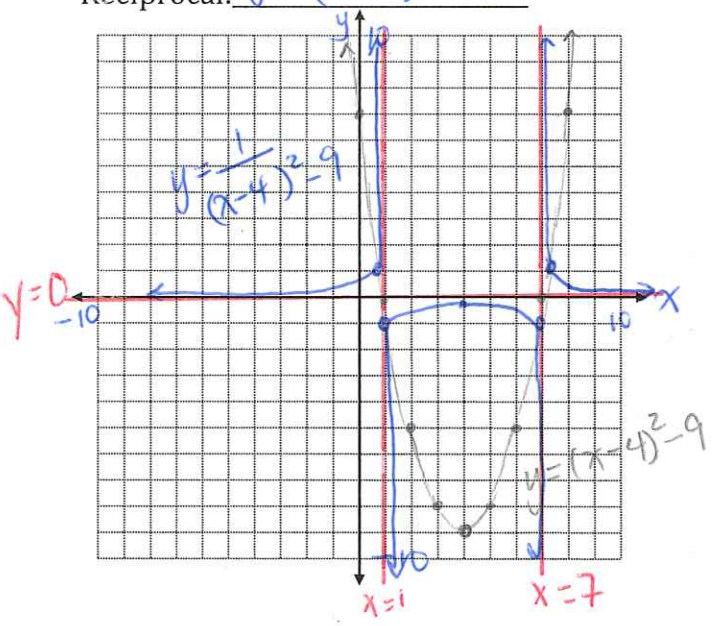
For example, the reciprocal of the function $y=x^2-9$ is $y=\frac{1}{x^2-9}$, $x\neq\pm 3$

Today, we will focus specifically on quadratic reciprocal functions.

Example: Graph each function and its reciprocal. For each reciprocal function, label any asymptotes, invariant points, and intercepts.

a) $y=(x-4)^2-9$

Reciprocal: $y=\frac{1}{(x-4)^2-9}$



Vertical Asymptote: $x=1$, $x=7$

Horizontal Asymptote: $y=0$

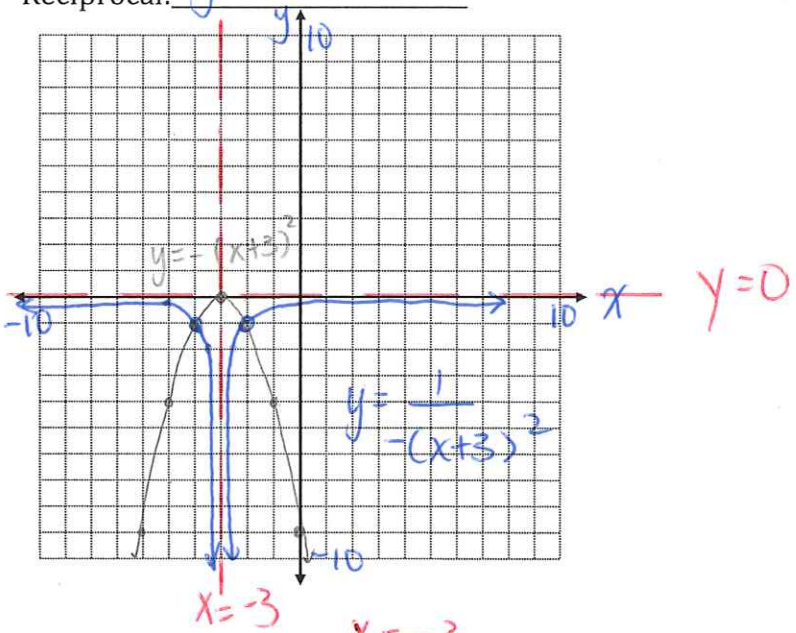
Invariant Points: (0.84, 1), (7.16, 1), (1.17, -1), (6.83, -1)

x-intercept: none

y-intercept: $(0, 1/7)$

b) $y=-(x+3)^2$

Reciprocal: $y=\frac{1}{-(x+3)^2}$



Vertical Asymptote: $x=-3$

Horizontal Asymptote: $y=0$

Invariant Points: $(-4,-1)$, $(-2,-1)$

x-intercept: none

y-intercept: $(0, -1/9)$

c) $y = x^2 + 1$

Reciprocal: $y = \frac{1}{x^2+1}$

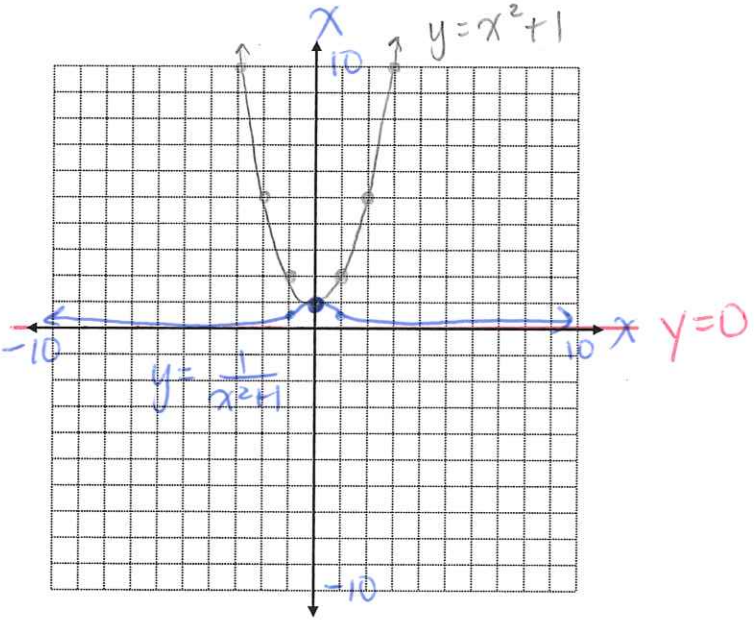
Vertical Asymptote: none

Horizontal Asymptote: $y = 0$

Invariant Points: $(0, 1)$

x-intercept: none

y-intercept: $(0, 1)$



Example: For each function:

i) Determine the zero (x–intercept).

ii) State the equation of the reciprocal function, its non-permissible values, and its asymptotes.

a) $y = x^2 - 9$

i) zero ($y = 0$)

$0 = x^2 - 9$

$x^2 = 9$

$x = \pm 3$

ii) reciprocal: $y = \frac{1}{x^2-9}$

$x \neq \pm 3$

asymptotes:

$x = -3, x = 3, y = 0$

b) $y = x^2 + 2x - 3$

i) zero ($y = 0$)

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$x = -3, 1$

ii) reciprocal: $y = \frac{1}{x^2+2x-3} = \frac{1}{(x+3)(x-1)}$

$x \neq -3, 1$

asymptotes: ~~$x \neq -3, 1, y = 0$~~

$x = -3, x = 1, y = 0$