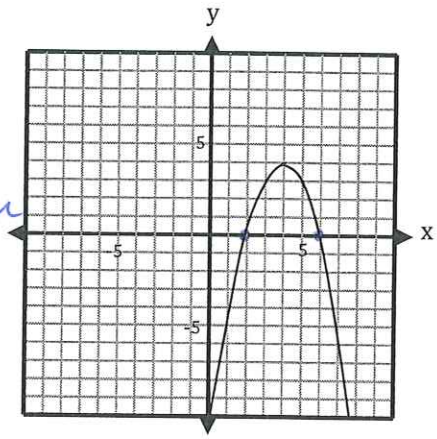
9.2 Quadratic Inequalities in One Variable (Part 1)

Example: The graph of $y = -x^2 + 8x - 12$ is shown.
Use the graph to solve the following inequalities.



a) $-x^2 + 8x - 12 \geq 0$

We need to find where the function is greater than or equal to zero (ie. on OR above the x-axis)

$2 \leq x \leq 6$
OR
$[2, 6]$

b) $-x^2 + 8x - 12 < 0$

We need to find where the function is less than zero (negative)

$x < 2$ or $x > 6$
OR
$(-\infty, 2) \cup (6, \infty)$

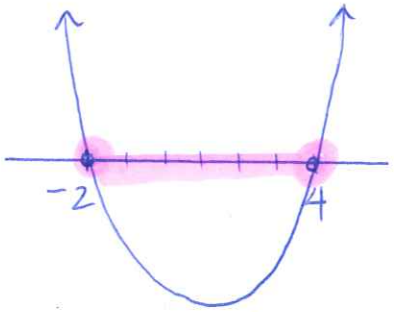
Notice that $y = -x^2 + 8x - 12 = -(x-2)(x-6)$

Example: Solve each inequality by graphing.

a) $x^2 - 2x - 8 \leq 0$

Factor:
$(x-4)(x+2) \leq 0$

- The parabola meets the x-axis at $x = -2$ and 4 (the roots)
- This parabola opens "up"
- Sketch:

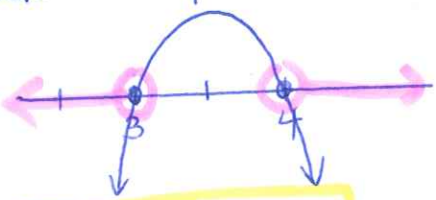


Where is the graph less than or equal to zero?

$-2 \leq x \leq 4$

b) $-x^2 + 7x < 12$ ← Not equal to

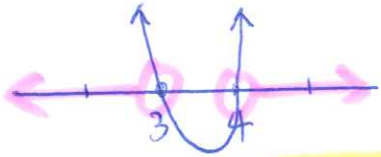
$-x^2 + 7x - 12 < 0$
$-(x^2 - 7x + 12) < 0$
$-(x-3)(x-4) < 0$ Roots: 3, 4
Parabola opens "down"



$x < 3$ or $x > 4$

OR

$-x^2 + 7x - 12 < 0$
$x^2 - 7x + 12 > 0$
$(x-3)(x-4) > 0$



$x < 3$ or $x > 4$

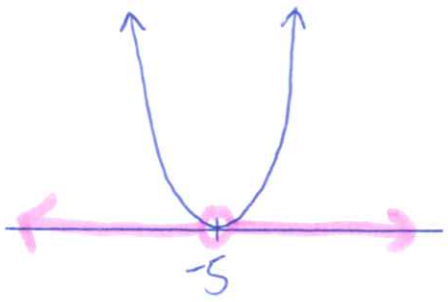
c) $x^2+10x+25>0$

$(x+5)^2>0$

One root: -5

This parabola meets the x-axis at one point, $x=-5$. The parabola opens "up."

The vertex must be at $(-5,0)$



Where is this parabola greater than zero?

Everywhere except for at $x=-5$

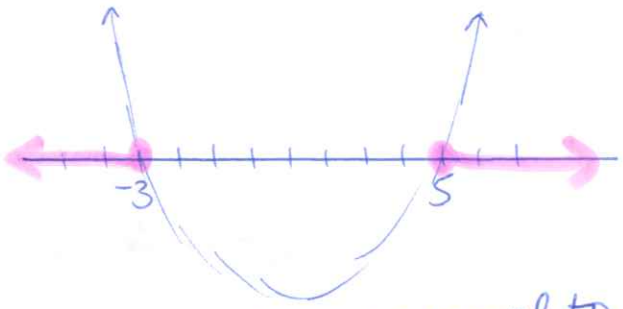
$x\neq -5$ OR $x<-5$ or $x>-5$

OR $(-\infty,-5)\cup(-5,\infty)$

Example: Write an inequality that has each given solution.

a) $x\leq -3$ *or* $x\geq 5$

The roots are at -3 and 5



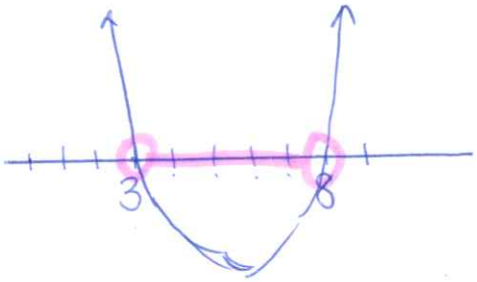
greater than or equal to zero.

$(x+3)(x-5)\geq 0$

$x^2-2x-15\geq 0$

b) $3<x<8$

The roots are 3 and 8



less than zero

$(x-3)(x-8)<0$

$x^2-11x+24<0$