

### The General Term of an Arithmetic Sequence

Insert 2 numbers between 2 and 20 so that the four numbers form an arithmetic sequence.

$$2, \underline{8}, \underline{14}, 20$$

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$$20 - 2 = 18$$

$$18 \div 3 = 6$$

This leads to an equation for the general term:

$$t_n = a + (n - 1)d$$

where  $a$  = the first term

$n$  = number of terms

$d$  = common difference

Ex. 1) Given the sequence 2, 9, 16, ...

a) Determine a simplified equation for the general term.

$$a = 2$$

$$d = 7$$

$$t_n = 2 + (n - 1)7$$

$$= 7n - 5$$

b) Determine  $t_{30}$ .

$$n = 30$$

$$t_{30} = 2 + (30 - 1)7$$

$$= 205$$

c) Determine  $t_{100}$ .

$$n = 100$$

$$t_{100} = 2 + (100 - 1)7$$

$$= 695$$

Ex. 2) -65 is a term in the sequence 19, 13, 7, .... Which term is it?

$$a = 19$$

$$d = -6$$

$$n = ?$$

$$t_n = -65$$

$$t_n = a + (n - 1)d$$

$$-65 = 19 + (n - 1)(-6)$$

$$-84 = -6(n - 1)$$

$$14 = n - 1$$

$$n = 15$$

Ex. 3) In an arithmetic sequence, the 5<sup>th</sup> term is 53 and the 12<sup>th</sup> term is 102.

a) What is the first term?

$$t_5 = 53$$

$$t_{12} = 102$$

$$a = ?$$

$$53, \underbrace{\quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad}_{53 + 7d = 102}, 102$$

$$53 + 7d = 102$$

$$7d = 49$$

$$d = 49$$

$$t_n = a + (n-1)d$$

$$53 = a + (5-1)(49)$$

$$53 = a + 4(49)$$

$$53 = a + 196$$

$$\boxed{a = -143}$$

b) Write the general term.

$$t_n = -143 + (n-1)(49)$$

$$= -143 + 49n - 49$$

$$= 49n - 192$$

c) How many terms in the sequence are less than 150?

$$t_n = 150 \quad n = ?$$

$$150 = -143 + (n-1)(49)$$

$$293 = (n-1)(49)$$

$$5.9796 = n-1$$

$$n = 6.9796$$

6 terms