

The Sum of an Arithmetic Series

Determine the sum of the numbers from 1 to 100.

$$1 + 2 + 3 + 4 + \dots + 96 + 97 + 98 + 99 + 100$$

50 pairs of 101

$$\text{So } 50(101) = 5050$$

What difference do you see between the problem above and the sequences we worked with previously?

We are adding the terms

series: the sum of a sequence

arithmetic series: a series whose terms are separated by a common difference
eg. $2 + 6 + 10 + \dots$

The sum of the first n terms of an arithmetic series is given by the following formula:

$$S_n = \frac{n}{2}(a + t_n)$$

Ex. 1) Determine the sum of the first 30 terms of the series $3 + 5 + 7 + 9 + \dots$

$$n = 30 \quad d = 2$$

$$a = 3$$

$$t_{30} = ?$$

$$t_{30} = 3 + (30 - 1)2$$

$$= 61$$

$$S_{30} = \frac{30}{2}(3 + 61)$$

$$= 960$$

Ex. 2) Determine the sum of the first 50 terms of the series $1 + 6 + 11 + \dots$

$$\begin{aligned} a &= 1 \\ d &= 5 \\ n &= 50 \\ t_{50} &= 1 + (50-1)5 \\ &= 264 \end{aligned}$$

$$\begin{aligned} S_6 &= \frac{50(1+264)}{2} \\ &= 6625 \end{aligned}$$

Ex. 3) Determine the sum of the series $5 + 9 + 13 + 17 + \dots + 65$.

$$\begin{aligned} a &= 5 \\ d &= 4 \\ t_n &= 65 \\ n &= ? \\ 65 &= 5 + (n-1)4 \\ 60 &= 4(n-1) \\ 15 &= n-1 \\ n &= 16 \end{aligned}$$

$$\begin{aligned} S_{16} &= \frac{16(5+65)}{2} \\ &= 560 \end{aligned}$$

Ex. 4) Determine the sum of the series $17 + 9 + 1 + \dots + (-95)$.

$$\begin{aligned} a &= 17 \\ d &= (-8) \\ t_n &= -95 \\ n &= ? \end{aligned}$$

$$\begin{aligned} S_{15} &= \frac{15(17-95)}{2} \\ &= -585 \end{aligned}$$

$$\begin{aligned} -95 &= 17 + (n-1)(-8) \\ -112 &= -8(n-1) \\ 14 &= n-1 \\ n &= 15 \end{aligned}$$