Chapter 3 Review

1. Explain how the graph of each function can be obtained from the graph of $f(x) = x^2$. For each graph, identify the direction of opening, whether it has a maximum or a minimum value, and the range.

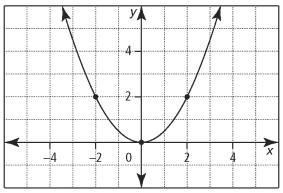
a) $f(x) = 3x^2$ **b)** $f(x) = -5x^2$ **c)** $f(x) = x^2 + 8$ **d)** $f(x) = x^2 - 5$

2. Explain how the graphs of each pair of functions are related. Sketch the graph of the second function in each pair, and determine its vertex, axis of symmetry, domain, range, and any intercepts.

a) $y = x^2$ and $y = (x - 6)^2$ **b)** $y = x^2$ and $y = (x + 1)^2$

c) $y = x^2$ and $y = (x + 4)^2 + 3$ **d)** $y = x^2$ and $y = (x - 2)^2 - 1$

3. **a)** Write a quadratic function in vertex form for the parabola shown on the graph.



b) Suppose the parabola is reflected about the x-axis. Write the quadratic function in vertex form of the new parabola.

c) Suppose the parabola in the graph is translated 6 units to the left. Write the quadratic function in vertex form of the new parabola.

d) Suppose the parabola in the graph is translated 3 units down. Write the quadratic function in vertex form of the new parabola.

5. Without graphing each function, identify the location of its vertex and axis of symmetry, direction of opening, maximum or minimum value, domain, range, and the number of x-intercepts.

a) $y = 3(x - 5)^2 + 1$ **b)** $y = -\frac{1}{2}(x + 2)^2$ **c)** $y = (x + 4)^2 - 5$ **d)** $y = -5x^2 + 3$

6. Determine a quadratic function in vertex form that has the given characteristics.

a) its vertex at (2, 0) and passes through the point (1, 3)

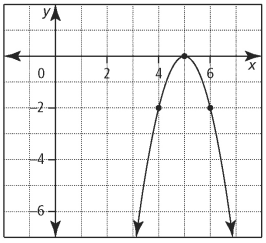
b) its vertex at (–2, 3) and passes through the point (–1, 1)

c) its vertex at (3, –2) and has an x-intercept of (5, 0)

d) its vertex at (4, 1) and has a y-intercept of (0, –15)

7. Determine a quadratic function in vertex form for each parabola.

a)



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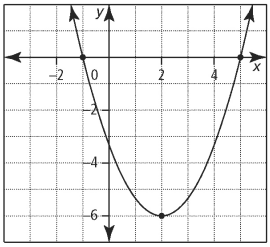


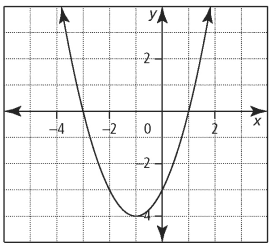
8. Which functions are quadratic? Explain why.

a) $y = x^2 - 15x$ **b)** $f(x) = (x + 4)(x - 4)$ **c)** $h(t) = -4.9t^2 + 400$ **d)** $V(w) = w(w + 3)(w - 1)$

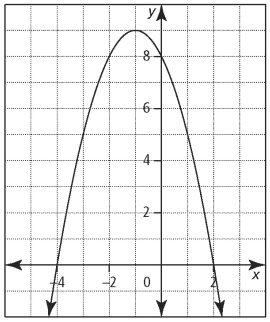
9. For each graph, identify the following:

- the coordinates of the vertex
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the direction of opening
- the maximum or minimum value
- the domain and range

a)



b)



11. Sketch the graph of each function. For each graph, determine

- the coordinates of the vertex
- the equation of the axis of symmetry
- the x-intercepts and y-intercept
- the direction of opening
- the maximum or minimum value
- the domain and range

a) $y = x^2 - 8x + 15$ **b)** $f(x) = -(x + 1)(x + 7)$
c) $y = x^2 - 4x$ **d)** $h(t) = 10t - 5t^2$

12. A farmer has 200 m of fencing material to enclose a rectangular field adjacent to a river.

a) Write a function that can be used to represent the area of the field.

b) Determine the maximum area of the field.

c) Determine the dimensions of the region that give the maximum area.

13. A projectile is fired out of a cannon at 105 m/s from a 100-m cliff. The function that models the height, h, of the trajectory in relation to time, t, is $h(t) = -5t^2 + 105t + 100$.

a) Determine the h-intercept of the function. What does the h-intercept represent?

b) Determine the t-intercept of the function. What does the t-intercept represent?

c) Determine the maximum height of the projectile and when it occurs.

15. Write each function in vertex form by completing the square. Use your answer to identify the vertex of the function.

a) $y = x^2 + 2x - 4$ **b)** $y = x^2 - 6x + 13$ **c)** $y = x^2 + 8x + 6$ **d)** $y = x^2 + 24x + 54$

16. Convert each function to the form $y = a(x - p)^2 + q$ by completing the square.

a) $y = 3x^2 - 12x + 13$ **b)** $y = -2x^2 - 20x - 56$ **c)** $y = 6x^2 - 48x$ **d)** $y = -4x^2 - 56x - 196$

17. Write each function in vertex form. Determine the maximum or minimum of each function and the x-value at which it occurs. Then, sketch a graph of the function.

a) $y = x^2 + 6x + 4$ **b)** $y = 2x^2 - 16x + 31$ **c)** $y = -3x^2 - 12x - 7$ **d)** $y = -x^2 + 18x$

18. Convert each function to the form $y = a(x - p)^2 + q$. State the coordinates of the vertex, axis of symmetry, maximum or minimum value, domain, and range.

a) $y = x^2 + 10x + 16$ **b)** $y = -3x^2 - 6x + 3$ **c)** $y = 2x^2 + 30x + 117$

19. If a farmer harvests his crop today, he will have 1200 bushels worth \$6 per bushel. Every week he waits, the crop yield increases by 100 bushels, but the price drops 30¢ per bushel.

a) What quadratic function can be used to model this situation?

b) When should the farmer harvest his crop to maximize his revenue? What is the maximum revenue?

Answers

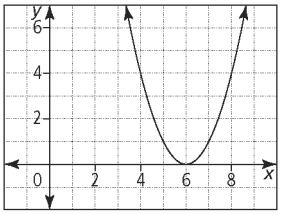
1. a) The graph can be obtained by applying a change in width about the x-axis by a factor of 3. The graph opens upward, has a minimum y-value of 0, and the range is $y \geq 0$.

b) The graph can be obtained by applying a change in width about the x-axis by a factor of 5, and then a reflection in the x-axis. The graph opens downward, has a maximum y-value of 0, and the range is $y \leq 0$.

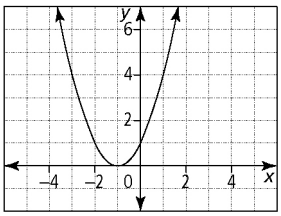
c) The graph can be obtained by applying a vertical translation up 8 units. The graph opens upward, has a minimum y-value of 8, and the range is $y \geq 8$.

d) The graph can be obtained by applying a vertical translation down 5 units. The graph opens upward, has a minimum y-value of –5, and the range is $y \geq -5$.

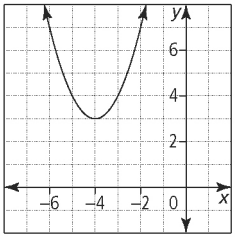
2. a) The graph of $y = (x - 6)^2$ can be obtained from $y = x^2$ by applying a horizontal translation 6 units to the right.
vertex: (6, 0); axis of symmetry: $x = 6$; domain: $x \in \mathbb{R}$; range: $y \geq 0$; x-intercept: $x = 6$; y-intercept: $y = 36$



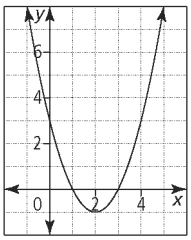
b) The graph of $y = (x + 1)^2$ can be obtained by applying a horizontal translation 1 unit to the left.
vertex: (–1, 0); axis of symmetry: $x = -1$; domain: $x \in$ R; range: $y \geq 0$; x-intercept: $x = -1$; y-intercept: $y = 1$



c) The graph of $y = (x + 4)^2 + 3$ can be obtained by applying a horizontal translation 4 units to the left and a vertical translation 3 units up.
vertex: (–4, 3); axis of symmetry: $x = -4$; domain: $x \in$ R; range: $y \geq 3$; x-intercept: none; y-intercept: $y = 19$



d) The graph of $y = (x - 2)^2 - 1$ can be obtained by applying a horizontal translation 2 units to the right and a vertical translation 1 unit down.
vertex: (2, –1); axis of symmetry: $x = 2$; domain: $x \in$ R; range: $y \geq -1$; x-intercepts: $x = 1$ and 3; y-intercept: $y = 3$



3. a) $y = 0.5x^2$ **b)** $y = -0.5x^2$ **c)** $y = 0.5(x + 6)^2$ **d)** $y = 0.5x^2 - 3$

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5.

	a)	**b)**	**c)**	**d)**
Vertex	(5, 1)	(–2, 0)	(–4, –5)	(0, 3)
Axis of symmetry	$x = 5$	$x = -2$	$x = -4$	$x = 0$
Direction	upward	downward	upward	downward
Max/min	min $y = 1$	max $y = 0$	min $y = -5$	max $y = 3$
Domain	$x \in$ R	$x \in$ R	$x \in$ R	$x \in$ R
Range	$y \geq 1$	$y \leq 0$	$y \geq -5$	$y \leq 3$
Number of *x*-intercepts	0	1	2	2

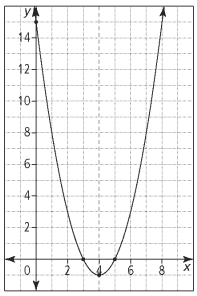
6. a) $y = 3(x - 2)^2$ **b)** $y = -2(x + 2)^2 + 3$ **c)** $y = \frac{1}{2}(x - 3)^2 - 2$ **d)** $y = -1(x - 4)^2 + 1$

7. a) $f(x) = -2(x - 5)^2$ **b)** $f(x) = \frac{2}{3}(x - 2)^2 - 6$

8. a) Yes. The function fits the standard form of a quadratic function with $a = 1$, $b = -15$, and $c = 0$.
b) $y = x^2 - 16$ Yes. The function fits the standard form of a quadratic function with $a = 1$, $b = 0$, and $c = -16$.
c) Yes. The function fits the standard form of a quadratic function with $a = -4.9$, $b = 0$, and $c = 400$.
d) No. The function does not fit the standard form of a quadratic function.

9.

	a)	**b)**
Vertex	(–1, –4)	(–1, 9)
Axis of symmetry	$x = -1$	$x = -1$
***x*-intercepts**	–3 and 1	–4 and 2
***y*-intercept**	–3	8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