

Compound Interest: Future Value

Guaranteed investment certificates (GICs) can earn either simple or compound interest. If a GIC earns simple interest annually, the same amount of interest is earned every year. If a GIC earns **compound interest** annually, the interest at the end of the first year is earned on the principal, but the interest at the end of the second year is earned on principal plus the interest from the first year. Each year after that, the interest is earned on the principal plus all the accumulated interest from previous years.

Investigate: Both Ewan and Rena received a \$1000 prize in a story-writing contest.

- Ewan bought a \$1000 simple interest GIC with his prize money. It has a 5-year term and earns 3.6% paid annually.
- Rena bought a \$1000 compound interest GIC with her prize money. It also has a 5-year term and earns 3.6% paid annually.

How do the future values of Ewan's and Rena's investments compare at maturity?

Ewan:

$$\begin{aligned} & \text{present value} + \text{interest} \\ & = 1000 + 1000(0.036)(5) \\ & = \$1180 \end{aligned}$$

↑ interest rate ↑ years

Rena:

$$\begin{aligned} \text{Year 1: } & 1000 + 1000(0.036) = 1036 \\ \text{Year 2: } & 1036 + 1036(0.036) = 1073.30 \\ \text{Year 3: } & 1073.30 + 1073.3(0.036) \\ & = \$1111.94 \quad \begin{matrix} \uparrow \\ \text{rounded} \\ (\$) \end{matrix} \\ \text{Year 4: } & 1111.94 + 1111.94(0.036) \\ & = \$1151.97 \\ \text{Year 5: } & 1151.97 + 1151.97(0.036) \\ & = \$1193.44 \end{aligned}$$

Rena earns \$13.44 more.

Compound Interest:

$$A = P(1 + i)^n$$

↑ present amount ↑ interest rate ← # of compounding periods

future amount

Example: Sydney has \$6500 to deposit into a savings account. She intends to keep the account for 4 years. Compare the following choices. Which account should she choose?

A. 5.1% compounded annually \leftarrow once a year

$$A = 6500(1 + 0.051)^4 = 7930.93$$

B. 5.1% compounded semi-annually \leftarrow twice a year

$$A = 6500 \left(1 + \frac{0.051}{2}\right)^{4(2)} \\ = 7950.58$$

C. 5.1% compounded quarterly \leftarrow 4 times a year

$$A = 6500 \left(1 + \frac{0.051}{4}\right)^{4(4)} \\ = 7960.67$$

D. 5.1% compounded monthly \leftarrow 12 times a year

$$A = 6500 \left(1 + \frac{0.051}{12}\right)^{4(12)} \\ = 7967.49$$

Choose Account D.

The Rule of 72 is a simple formula for estimating the doubling time of an investment.

Divide 72 by your interest rate to find out how long it will take to double your investment.

Estimate how long it will take an investment of \$5000 to double in a GIC that earns 6% compounded annually.

$$72 \div 6 = 12 \text{ years} \quad \text{check: } A = 5000(1 + 0.06)^{12} \\ = 10060.98 \\ \text{(very close!)}$$