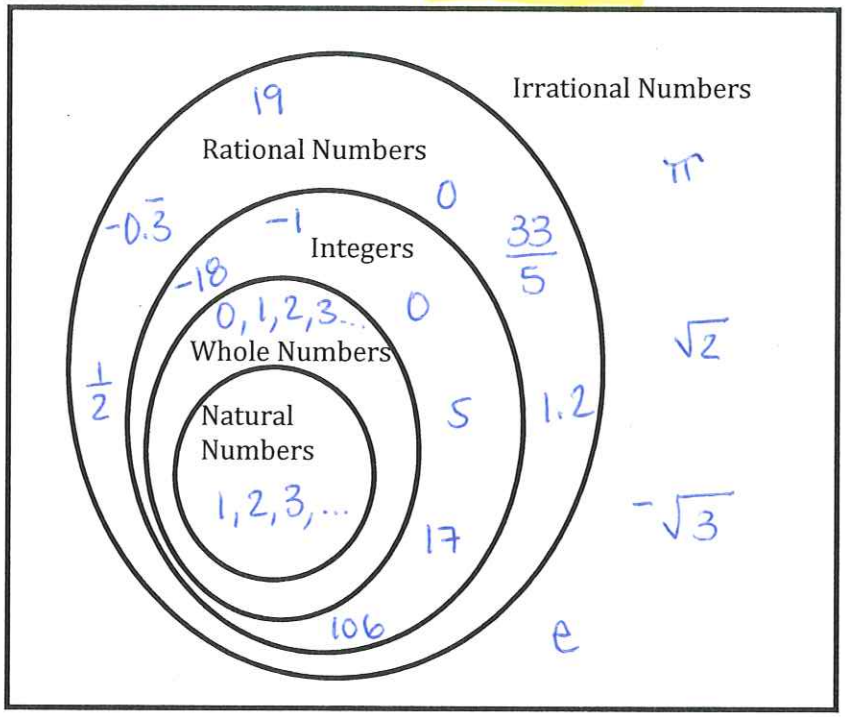
Pre-Calculus 11
Roots and Powers

Rational Numbers, Irrational Numbers and Radicals

A rational number can be written in the form $\frac{m}{n}$ where m and n are integers and $n \neq 0$. This includes integers, fractions, terminating decimals and repeating decimals.

An irrational number is a real number that is not rational. The decimal representation of an irrational number neither terminates nor repeats.

The following Venn Diagram represents the set of **Real Numbers.**



Example: Determine whether each number is rational or irrational. Explain.

I a) $\sqrt{8} = 2.828427125...$

R b) $\sqrt[3]{8} = 2$

R c) $\sqrt{\frac{4}{9}} = \frac{2}{3}$

R d) $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$

R e) $\sqrt[3]{-8} = -2$

I f) $\sqrt[3]{-4} = -1.587401052$.

R g) $\sqrt{0.81} = \sqrt{\frac{81}{100}} = \frac{9}{10}$

I g) $\sqrt[3]{0.8} = 0.9283177667$

All of the rational numbers can be expressed as fractions.

Multiplication Property of Radicals:	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, where a and b are real numbers and n is a natural number.

Example: Simplify each radical without a calculator.

b) $\sqrt{63} = \sqrt{9 \cdot 7}$
$= \sqrt{9} \cdot \sqrt{7}$
$= 3\sqrt{7}$

b) $\sqrt{24} = \sqrt{4 \cdot 6}$
$= 2\sqrt{6}$

c) $\sqrt{80} = \sqrt{8 \cdot 10}$
$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$
$= 4\sqrt{5}$

d) $\sqrt{30} = \sqrt{3 \cdot 10}$
$= \sqrt{2 \cdot 3 \cdot 5}$
$= \sqrt{30}$

e) $\sqrt[3]{108} = \sqrt[3]{2 \cdot 54}$
$= \sqrt[3]{2 \cdot 6 \cdot 9}$
$= \sqrt[3]{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$
$= 3\sqrt[3]{4}$

f) $\sqrt[3]{144} = \sqrt[3]{12 \cdot 12}$
$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$
$= 2\sqrt[3]{18}$

g) $\sqrt[3]{100} = \sqrt[3]{10 \cdot 10}$
$= \sqrt[3]{2 \cdot 5 \cdot 2 \cdot 5}$
$= \sqrt[3]{100}$

g) $\sqrt[4]{128} = \sqrt[4]{2 \cdot 64}$
$= \sqrt[4]{2 \cdot 8 \cdot 8}$
$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$
$= 2\sqrt[4]{8}$

Example: Write each mixed radical as an entire radical without using a calculator.

a) $5\sqrt{3} = \sqrt{5 \cdot 5 \cdot 3}$
$= \sqrt{125}$

b) $6\sqrt{2} = \sqrt{6 \cdot 6 \cdot 2}$
$= \sqrt{72}$

c) $2\sqrt[3]{5} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5}$
$= \sqrt[3]{40}$

d) $2\sqrt[4]{10} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 10}$
$= \sqrt[4]{160}$