

## Fractional Exponents and Radicals

### Recall the Exponent Laws:

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^m)^n = a^{mn}$
$(ab)^n = a^n b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$a^0 = 1$ if $a \neq 0$

Complete the following statements:

$$5^{1/2} \cdot 5^{1/2} = 5^{1/2+1/2}$$

$$= 5^1$$

$$= 5 \quad \leftarrow \text{same} \quad \rightarrow = 5$$

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{5 \cdot 5}$$

$$= 5$$

Complete the following statements:

$$5^{1/3} \cdot 5^{1/3} \cdot 5^{1/3} = 5^{1/3+1/3+1/3}$$

$$= 5^1$$

$$= 5 \quad \leftarrow \text{same} \quad \rightarrow = 5$$

$$\sqrt[3]{5} \cdot \sqrt[3]{5} \cdot \sqrt[3]{5} = \sqrt[3]{5 \cdot 5 \cdot 5}$$

$$= 5$$

When  $n$  is a natural number and  $a$  is a rational number,  $a^{1/n} = \sqrt[n]{a}$

Example: Express as a radical, then evaluate without a calculator.

a)  $100^{1/2} = \sqrt{100}$   
 $= 10$

b)  $27^{1/3} = \sqrt[3]{27}$   
 $= 3$

c)  $(-64)^{1/3} = \sqrt[3]{-64}$   
 $= -4$

d)  $\left(\frac{4}{9}\right)^{1/2} = \sqrt{\frac{4}{9}}$   
 $= \frac{2}{3}$

e)  $0.81^{1/2} = \sqrt{\frac{81}{100}}$   
 $= \frac{9}{10}$

f)  $-81^{1/4} = -\sqrt[4]{81}$   
 $= -3$

When  $m$  and  $n$  are natural numbers and  $a$  is a rational number,

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(a^m\right)^{1/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Example: Write each power as a radical and then evaluate without a calculator.

$$\begin{aligned} \text{a) } 32^{3/5} &= \left(\sqrt[5]{32}\right)^3 \\ &= 2^3 \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{b) } (-27)^{2/3} &= \left(\sqrt[3]{-27}\right)^2 \\ &= (-3)^2 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{c) } 81^{0.75} &= 81^{3/4} \\ &= \left(\sqrt[4]{81}\right)^3 \\ &= 3^3 \\ &= 27 \end{aligned}$$

$$\begin{aligned} \text{d) } 0.01^{3/2} &= \left(\frac{1}{100}\right)^{3/2} \\ &= \left(\sqrt{\frac{1}{100}}\right)^3 \\ &= \left(\frac{1}{10}\right)^3 = \frac{1}{1000} \end{aligned}$$

$$\begin{aligned} \text{e) } \left(\frac{16}{81}\right)^{3/4} &= \left(\sqrt[4]{\frac{16}{81}}\right)^3 \\ &= \left(\frac{2}{3}\right)^3 \\ &= \frac{8}{27} \end{aligned}$$

$$\begin{aligned} \text{f) } -25^{1.5} &= -25^{3/2} \\ &= -\left(\sqrt{25}\right)^3 \\ &= -(5)^3 \\ &= -125 \end{aligned}$$

Example: Write each of the following in exponential form.

$$\text{a) } \left(\sqrt[4]{2}\right)^5 = 2^{5/4}$$

$$\text{b) } \sqrt{7^3} = 7^{3/2}$$