

Negative Exponents and Reciprocals

Consider the expression $\frac{5^0}{5^2}$.

- a) Simplify the expression using the division law for exponents. $\frac{5^0}{5^2} = 5^{0-2} = 5^{-2} \rightarrow 5^{-2} = \frac{1}{25}$
- b) Evaluate the expression and write your answer as a fraction. $\frac{5^0}{5^2} = \frac{1}{5^2} = \frac{1}{25}$

When 'a' is a non-zero number and 'n' is a rational number, a^{-n} is the reciprocal of a^n .

That is, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$, if $a \neq 0$.

Example: Write each power with a positive exponent.

a) $6^{-2} = \frac{1}{6^2}$ b) $(-8)^{-3} = \frac{1}{(-8)^3}$ c) $\left(\frac{1}{4}\right)^{-2} = 4^2$

d) $\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^3$ e) $\frac{1}{8^{-2}} = 8^2$

Example: Evaluate without using a calculator.

a) $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$ b) $(-4)^{-3} = \frac{1}{(-4)^3} = -\frac{1}{64}$ c) $\left(\frac{4}{9}\right)^{-1} = \left(\frac{9}{4}\right)^1 = \frac{9}{4}$

d) $\left(\frac{1}{2}\right)^{-4} = \left(\frac{2}{1}\right)^4 = 16$ e) $0.5^{-3} = \left(\frac{1}{2}\right)^{-3} = \left(\frac{2}{1}\right)^3 = 8$ f) $\frac{1}{4^{-2}} = 4^2 = 16$

You can combine the negative exponent law with the rational exponent law from last class to answer the following questions.

Recall:

When m and n are natural numbers and a is a rational number, $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Example: Evaluate without using a calculator.

$$\begin{aligned} \text{a) } 9^{-1/2} &= \frac{1}{9^{1/2}} \\ &= \frac{1}{\sqrt{9}} \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } 16^{-5/4} &= \frac{1}{16^{5/4}} \\ &= \frac{1}{(\sqrt[4]{16})^5} \\ &= \frac{1}{(2)^5} \\ &= \frac{1}{32} \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{9}{16}\right)^{-3/2} &= \left(\frac{16}{9}\right)^{3/2} \\ &= \left(\sqrt{\frac{16}{9}}\right)^3 \\ &= \left(\frac{4}{3}\right)^3 \\ &= \frac{64}{27} \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{1}{27}\right)^{-2/3} &= 27^{2/3} \\ &= (\sqrt[3]{27})^2 \\ &= 3^2 \\ &= 9 \end{aligned}$$

Example: Use your calculator to evaluate $0.155(0.3)^{-7/6}$.

$$= 0.6314760751$$