Pre-Calculus 11

Roots & Powers

Supplemental Package

Discuss the Ideas

1. How do you determine whether a radical represents a rational or an irrational number? Use examples to explain.

2. How can you determine whether the decimal form of a radical represents its exact value?

Exercises

A

3. Tell whether each number is rational or irrational.
   a) $\sqrt{12}$ b) $\sqrt[4]{16}$
   c) $\sqrt[3]{-100}$ d) $\sqrt{\frac{4}{9}}$
   e) $\sqrt{1.25}$ f) 1.25

4. Classify each number below as:
   a) a natural number b) an integer
   c) a rational number d) an irrational number
   $\frac{4}{3}$, $0.3\overline{4}$, -5, $\sqrt[4]{9}$, -2.1538, $\sqrt[3]{27}$, 7

B

5. a) Why are $\sqrt{49}$ and $\sqrt[4]{16}$ rational numbers?
   b) Why are $\sqrt{21}$ and $\sqrt[3]{36}$ irrational numbers?

6. Look at this calculator screen.

   a) Is the number 12.247 448 71 rational or irrational? Explain.
   b) Is the number $\sqrt{150}$ rational or irrational? Explain.

7. a) Sketch a diagram to represent the set of rational numbers and the set of irrational numbers.
   b) Write each number that follows in the correct set.
   $\frac{1}{2}$, $-\sqrt{3}$, $\sqrt{4}$, $\sqrt[4]{5}$, $-\frac{7}{6}$, $\sqrt[3]{8}$, 10.12, $-13.\overline{4}$, $\sqrt{0.15}$, $\sqrt{0.16}$, 17

8. For which numbers will the cube root be irrational? Use 2 different strategies to justify your answers.
   a) 8 b) 64 c) 30 d) 300

9. Sketch a number line for each irrational number and label its approximate location. Explain your reasoning.
   a) $\sqrt{5}$ b) $\sqrt[3]{12}$ c) $\sqrt[4]{25}$ d) $\sqrt[3]{-12}$

10. Use a number line to order the irrational numbers in each set from greatest to least.
    a) $\sqrt[3]{70}$, $\sqrt{50}$, $\sqrt[4]{100}$, $\sqrt[3]{400}$
    b) $\sqrt{89}$, $\sqrt[4]{250}$, $\sqrt[3]{-150}$, $\sqrt[3]{150}$

11. Use a number line to order these numbers from least to greatest. How can you verify your answer?
    $\sqrt{40}$, $\sqrt[3]{500}$, $\sqrt{98}$, $\sqrt[3]{98}$, $\sqrt{75}$, $\sqrt[3]{300}$

12. Use a number line to order these numbers from least to greatest. Identify which numbers are irrational and which are rational.
    $\frac{-14}{5}$, $\frac{123}{99}$, -2, $\sqrt[3]{-10}$, $\sqrt{4}$

13. How do you use irrational numbers when you calculate the length of the hypotenuse of a right triangle with legs 5 cm and 3 cm?

14. a) Which of the following statements are true? Explain your reasoning.
    i) All natural numbers are integers.
    ii) All integers are rational numbers.
    iii) All whole numbers are natural numbers.
    iv) All irrational numbers are roots.
    v) Some rational numbers are natural numbers.
    b) For each statement in part a that is false, provide examples to explain why.

15. Write a number that is:
    a) a rational number but not an integer
    b) a whole number but not a natural number
    c) an irrational number

Exercises

A

3. List all the perfect squares up to 400, and their square roots.

4. Write each radical in simplest form.

a) $\sqrt{8}$ b) $\sqrt{12}$ c) $\sqrt{32}$ d) $\sqrt{50}$

e) $\sqrt{18}$ f) $\sqrt{27}$ g) $\sqrt{48}$ h) $\sqrt{75}$

5. Write each mixed radical as an entire radical.

a) $5\sqrt{2}$ b) $6\sqrt{2}$ c) $7\sqrt{2}$ d) $8\sqrt{2}$

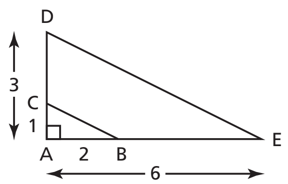
e) $5\sqrt{3}$ f) $6\sqrt{3}$ g) $7\sqrt{3}$ h) $8\sqrt{3}$

6. a) List all the perfect cubes up to 1000, and their cube roots.

b) List all the perfect fourth powers up to 1000, and their fourth roots.

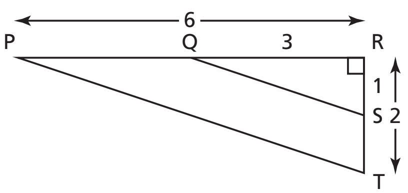
B

7. a) Use the diagram to explain why $\sqrt{45} = 3\sqrt{5}$.



b) Use algebra to verify that $\sqrt{45} = 3\sqrt{5}$.

8. a) Use the diagram to explain why $\sqrt{40} = 2\sqrt{10}$.



b) Use algebra to verify that $\sqrt{40} = 2\sqrt{10}$.

9. Explain why rewriting $\sqrt{50}$ as $\sqrt{25} \cdot \sqrt{2}$ helps you simplify $\sqrt{50}$, but rewriting $\sqrt{50}$ as $\sqrt{10} \cdot \sqrt{5}$ does not.

10. Write each radical in simplest form, if possible.

a) $\sqrt{90}$ b) $\sqrt{73}$ c) $\sqrt{108}$

d) $\sqrt{600}$ e) $\sqrt{54}$ f) $\sqrt{91}$

g) $\sqrt{28}$ h) $\sqrt{33}$ i) $\sqrt{112}$

11. Write each radical in simplest form, if possible.

a) $\sqrt[3]{16}$ b) $\sqrt[3]{81}$

c) $\sqrt[3]{256}$ d) $\sqrt[3]{128}$

e) $\sqrt[3]{60}$ f) $\sqrt[3]{192}$

g) $\sqrt[3]{135}$ h) $\sqrt[3]{100}$

i) $\sqrt[3]{500}$ j) $\sqrt[3]{375}$

12. Write each mixed radical as an entire radical.

a) $3\sqrt{2}$ b) $4\sqrt{2}$

c) $6\sqrt{5}$ d) $5\sqrt{6}$

e) $7\sqrt{7}$ f) $2\sqrt[3]{2}$

g) $3\sqrt[3]{3}$ h) $4\sqrt[3]{3}$

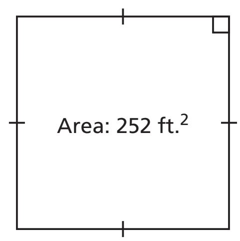
i) $5\sqrt[3]{2}$ j) $2\sqrt[3]{9}$

13. a) Can every mixed radical be expressed as an entire radical?

b) Can every entire radical be expressed as a mixed radical?

Give examples to support your answers.

14. Express the side length of this square as a radical in simplest form.



15. A cube has a volume of 200 cm^3. Write the edge length of the cube as a radical in simplest form.

16. A square has an area of 54 square inches. Determine the perimeter of the square. Write the answer as a radical in simplest form.

17. Write each radical in simplest form.

a) $\sqrt[4]{48}$ b) $\sqrt[4]{405}$

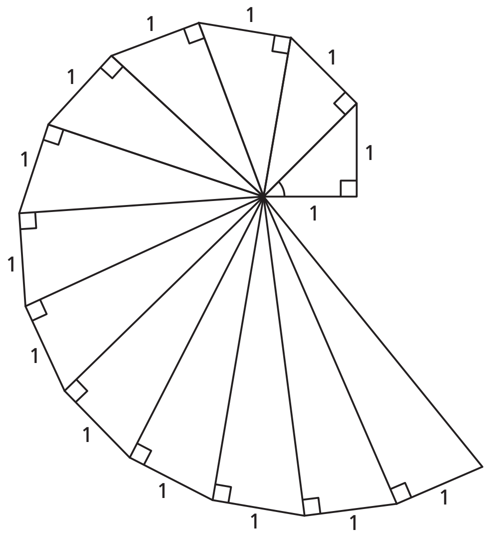
c) $\sqrt[4]{1250}$ d) $\sqrt[4]{176}$

18. Write each mixed radical as an entire radical.

a) $6\sqrt[4]{3}$ b) $7\sqrt[4]{2}$

c) $3\sqrt[5]{4}$ d) $4\sqrt[5]{3}$

19. The quilt on page 213 is made from right triangles. In Chapter 2, page 77, you determined the tangents of the angles at the centre of the spiral. The first triangle is a right isosceles triangle with legs 1 unit long. The hypotenuse of this triangle is one leg of the second triangle, with its other leg 1 unit long. This pattern continues.



a) Calculate the length of the hypotenuse of each triangle. Write each length as an entire radical.

b) **i)** What pattern do you see in the lengths?

ii) Use this pattern to predict the length of the hypotenuse of the 50th triangle.

iii) How many of the first 100 triangles have hypotenuse lengths that can be written as mixed radicals? Justify your answer.

20. Here is a student's solution for writing $8\sqrt[3]{2}$ as an entire radical.

$$8\sqrt[3]{2} = 8 \cdot \sqrt[3]{2}$$
$$= \sqrt[3]{2} \cdot \sqrt[3]{2}$$
$$= \sqrt[3]{2 \cdot 2}$$
$$= \sqrt[3]{4}$$

Identify an error the student made, then write the correct solution.

21. A student simplified $\sqrt{96}$ as shown:

$$\sqrt{96} = \sqrt{4} \cdot \sqrt{48}$$
$$= 2 \cdot \sqrt{48}$$
$$= 2 \cdot \sqrt{8} \cdot \sqrt{6}$$
$$= 2 \cdot 4 \cdot \sqrt{6}$$
$$= 8\sqrt{6}$$

Identify the errors the student made, then write a correct solution.

22. Arrange in order from greatest to least. What strategy did you use each time?

a) $9\sqrt{2}, 2\sqrt{6}, 8\sqrt{3}, 4\sqrt{5}, 6\sqrt{2}$

b) $4\sqrt{7}, 8\sqrt{3}, 2\sqrt{13}, 6\sqrt{5}$

c) $7\sqrt{3}, 9\sqrt{2}, 5\sqrt{6}, \sqrt{103}, 3\sqrt{17}$

23. Simplify the radicals in each list.
What patterns do you see in the results?
Write the next 2 radicals in each list.

a) $\sqrt{4}$, $\sqrt{400}$, $\sqrt{40\,000}$

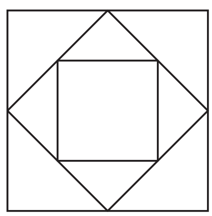
b) $\sqrt[3]{27}$, $\sqrt[3]{27\,000}$, $\sqrt[3]{27\,000\,000}$

c) $\sqrt{8}$, $\sqrt{800}$, $\sqrt{80\,000}$

d) $\sqrt[3]{24}$, $\sqrt[3]{24\,000}$, $\sqrt[3]{24\,000\,000}$

C

24. The largest square in this diagram has side length 8 cm. Calculate the side length and area of each of the two smaller squares. Write the radicals in simplest form.



25. Given that $\sqrt{2} \doteq 1.4142$, determine a decimal approximation for each radical, without using a calculator.

a) **i)** $\sqrt{200}$ **ii)** $\sqrt{20\,000}$

b) **i)** $\sqrt{8}$ **ii)** $\sqrt{18}$ **iii)** $\sqrt{32}$ **iv)** $\sqrt{50}$

Reflect

How do you use the index of a radical when you simplify a radical, and when you write a mixed radical as an entire radical? Use examples to support your explanation.

Discuss the Ideas

1. When a is a rational number and n is a natural number, what does $a^{\frac{1}{n}}$ represent?

2. When a is a rational number and m and n are natural numbers, what does $a^{\frac{m}{n}}$ represent?

Exercises

A

3. Evaluate each power without using a calculator.
   a) $16^{\frac{1}{2}}$ b) $36^{\frac{1}{2}}$ c) $64^{\frac{1}{3}}$
   d) $32^{\frac{1}{5}}$ e) $(-27)^{\frac{1}{3}}$ f) $(-1000)^{\frac{1}{3}}$

4. Evaluate each power without using a calculator.
   a) $100^{0.5}$ b) $81^{0.25}$
   c) $1024^{0.2}$ d) $(-32)^{0.2}$

5. Write each power as a radical.
   a) $36^{\frac{1}{3}}$ b) $48^{\frac{1}{2}}$ c) $(-30)^{\frac{1}{5}}$

6. Write each radical as a power.
   a) $\sqrt{39}$ b) $\sqrt[4]{90}$
   c) $\sqrt[3]{29}$ d) $\sqrt[5]{100}$

7. Evaluate each power without using a calculator.
   a) 8^0 b) $8^{\frac{1}{3}}$ c) $8^{\frac{2}{3}}$
   d) $8^{\frac{3}{3}}$ e) $8^{\frac{4}{3}}$ f) $8^{\frac{5}{3}}$

B

8. Write each power as a radical.
   a) $4^{\frac{2}{3}}$ b) $(-10)^{\frac{3}{5}}$ c) $2.3^{\frac{3}{2}}$

9. A cube has a volume of 350 cm^3. Write the edge length of the cube as a radical and as a power.

10. Write each power as a radical.
    a) $48^{\frac{2}{3}}$ b) $(-1.8)^{\frac{5}{3}}$ c) $\left(\frac{3}{8}\right)^{2.5}$
    d) $0.75^{0.75}$ e) $\left(-\frac{5}{9}\right)^{\frac{2}{5}}$ f) $1.25^{1.5}$

11. Write each radical as a power.
    a) $\sqrt{3.8^3}$ b) $\left(\sqrt[3]{-1.5}\right)^2$ c) $\sqrt[4]{\left(\frac{9}{5}\right)^5}$
    d) $\sqrt[3]{\left(\frac{3}{8}\right)^4}$ e) $\left(\sqrt{\frac{5}{4}}\right)^3$ f) $\sqrt[5]{(-2.5)^3}$

12. Evaluate each power without using a calculator.
    a) $9^{\frac{3}{2}}$ b) $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ c) $(-27)^{\frac{2}{3}}$
    d) $0.36^{1.5}$ e) $(-64)^{\frac{2}{3}}$ f) $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

13. Write an equivalent form for each number using a power with exponent $\frac{1}{2}$, then write the answer as a radical.
    a) 2 b) 4 c) 10 d) 3 e) 5

14. Write an equivalent form for each number using a power with exponent $\frac{1}{3}$, then write the answer as a radical.
    a) -1 b) 2 c) 3 d) -4 e) 4

15. Arrange these numbers in order from least to greatest. Describe your strategy.
    $\sqrt[3]{4}, 4^{\frac{3}{2}}, 4^2, \left(\frac{1}{4}\right)^{\frac{3}{2}}$

16. a) Evaluate.
    i) $16^{1.5}$ ii) $81^{0.75}$
    iii) $(-32)^{0.8}$ iv) $35^{0.5}$
    v) $1.21^{1.5}$ vi) $\left(\frac{3}{4}\right)^{0.6}$

    b) Which powers in part a could you have evaluated without a calculator? How can you tell before you evaluate?

17. The height, h metres, of a certain species of fir tree can be estimated from the formula $h = 35d^{\frac{2}{3}}$, where d metres is the diameter at the base. Use the formula to determine the approximate height of a fir tree with base diameter 3.2 m.

18. Here is a student's solution for evaluating a power.

$$1.96^{\frac{3}{2}} = \left(\sqrt[3]{1.96}\right)^2$$
$$= (1.2514\ldots)^2$$
$$= 1.5661\ldots$$

Identify the errors the student made. Write a correct solution.

19. A formula for the approximate surface area, SA square metres, of a person's body is $SA = 0.096m^{0.7}$, where m is the person's mass in kilograms. Calculate the surface area of a child with mass 40 kg.

20. Here is an expression for the percent of caffeine that remains in your body n hours after you drink a caffeine beverage:

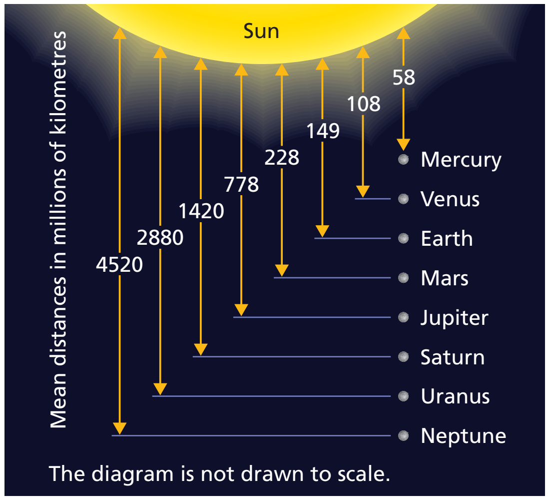
$100(0.5)^{\frac{n}{5}}$

a) Show that this expression and the expression on page 222 give the same result, to the nearest whole number, for the percent of caffeine that remains after $\frac{1}{2}$ h.

b) Use the expression above to determine the percent of caffeine that remains after 1.5 h.

c) After how many hours does 50% of the caffeine remain? Explain how you know.

21. In the late 1500s, Johannes Kepler developed a formula to calculate the time it takes each planet to orbit the sun (called the *period*). The formula is $T \doteq 0.2R^{\frac{3}{2}}$, where T is the period in Earth days and R is the mean distance from the planet to the sun in millions of kilometres.



The mean distance of Earth from the sun is about 149 million kilometres. The mean distance of Mars from the sun is about 228 million kilometres. Which planet has the longer period, Earth or Mars? Justify your answer.

C

22. Two students discussed the meaning of the statement $3.2^{4.2} = 132.3213...$

Luc said: It means 3.2 multiplied by itself 4.2 times is about 132.3213.

Karen said: No, you can't multiply a number 4.2 times. $3.2^{4.2}$ can be written as $3.2^{\frac{42}{10}}$. So the statement means that 42 factors, each equal to the tenth root of 3.2, multiplied together will equal about 132.3213.

Which student is correct? Explain.

Reflect

In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

What must be true about x for $x^{\frac{m}{n}}$ to be a rational number?

Discuss the Ideas

1. When m is an integer, describe the relationship between a^m and a^{-m}.
2. Why is there usually more than one way to determine the value of a power of the form $a^{-\frac{m}{n}}$? Use examples to justify your answer.

Exercises

A

3. Copy then complete each equation.

   a) $\frac{1}{5^4} = 5^{\square}$ b) $\left(-\frac{1}{2}\right)^{-3} = (-2)^{\square}$

   c) $\frac{1}{3^{\square}} = 3^2$ d) $\frac{1}{4^{-2}} = 4^{\square}$

4. Evaluate the powers in each pair without a calculator.

   a) 4^2 and 4^{-2} b) 2^4 and 2^{-4}

   c) 6^1 and 6^{-1} d) 4^3 and 4^{-3}

   Describe what is similar about the answers, and what is different.

5. Given that $2^{10} = 1024$, what is 2^{-10}?

6. Write each power with a positive exponent.

   a) 2^{-3} b) 3^{-5} c) $(-7)^{-2}$

7. Write each power with a positive exponent.

   a) $\left(\frac{1}{2}\right)^{-2}$ b) $\left(\frac{2}{3}\right)^{-3}$ c) $\left(-\frac{6}{5}\right)^{-4}$

8. Evaluate each power without using a calculator.

   a) 3^{-2} b) 2^{-4} c) $(-2)^{-5}$

   d) $\left(\frac{1}{3}\right)^{-3}$ e) $\left(-\frac{2}{3}\right)^{-2}$ f) $\frac{1}{5^{-3}}$

B

9. Evaluate each power without using a calculator.

   a) $4^{-\frac{1}{2}}$ b) $0.09^{-\frac{1}{2}}$

   c) $27^{-\frac{1}{3}}$ d) $(-64)^{-\frac{1}{3}}$

   e) $(-0.027)^{-\frac{2}{3}}$ f) $32^{-\frac{2}{5}}$

   g) $9^{-\frac{3}{2}}$ h) $0.04^{-\frac{3}{2}}$

10. Use a power with a negative exponent to write an equivalent form for each number.

    a) $\frac{1}{9}$ b) $\frac{1}{5}$ c) 4 d) -3

11. When you save money in a bank, the bank pays you *interest*. This interest is added to your investment and the resulting amount also earns interest. We say the interest *compounds*. Suppose you want an amount of \$3000 in 5 years. The interest rate for the savings account is 2.5% compounded annually. The money, P dollars, you must invest now is given by the formula: $P = 3000(1.025)^{-5}$. How much must you invest now to have \$3000 in 5 years?

12. Here is a student's solution for evaluating a power. Identify any errors in the solution. Write a correct solution.

$$\begin{aligned}\left(-\frac{64}{125}\right)^{-\frac{5}{3}} &= \left(\frac{64}{125}\right)^{\frac{5}{3}}\\ &= \left(\sqrt[3]{\frac{64}{125}}\right)^{5}\\ &= \left(\frac{4}{5}\right)^{5}\\ &= \frac{1024}{3125}\end{aligned}$$

13. Evaluate each power without using a calculator.

    a) $27^{-\frac{4}{3}}$ b) $16^{-1.5}$ c) $32^{-0.4}$

    d) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$ e) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ f) $\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

14. Michelle wants to invest enough money on January 1st to pay her nephew \$150 at the end of each year for the next 10 years. The savings account pays 3.2% compounded annually. The money, P dollars, that Michelle must invest today is given by the formula $P = \frac{150\left[1 - 1.032^{-10}\right]}{0.032}$. How much must Michelle invest on January 1st?

Example 4 Solving Problems Using the Exponent Laws

A sphere has volume 425 m^3.
What is the radius of the sphere to the nearest tenth of a metre?

SOLUTION

The volume V of a sphere with radius r is given by the formula: $V = \frac{4}{3}\pi r^3$. Substitute $V = 425$, then solve for r.

$$425 = \frac{4}{3}\pi r^3$$

Multiply each side by 3.

$$3(425) = 3\left(\frac{4}{3}\pi r^3\right)$$

$$1275 = 4\pi r^3$$

Divide each side by 4π.

$$\frac{1275}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\frac{1275}{4\pi} = r^3$$

To solve for r, take the cube root of each side by raising each side to the one-third power.

$$\left(\frac{1275}{4\pi}\right)^{\frac{1}{3}} = (r^3)^{\frac{1}{3}}$$

Use the power of a power law.

$$\left(\frac{1275}{4\pi}\right)^{\frac{1}{3}} = r$$

3ˣ√(1275/(4π))
4.664088405

$$r = 4.6640\ldots$$

The radius of the sphere is approximately 4.7 m.

CHECK YOUR UNDERSTANDING

4. A cone with height and radius equal has volume 18 cm^3. What are the radius and height of the cone to the nearest tenth of a centimetre?

[Answer: approximately 2.6 cm]

How do you know that the length of the radius is an irrational number?

Discuss the Ideas

1. Suppose you want to evaluate an algebraic expression for particular values of the variables. Why might it be helpful to simplify the expression first?

2. When you simplify an expression, how do you know which exponent law to apply first?

Exercises

A

3. Simplify.
a) $x^3 \cdot x^4$ **b)** $a^2 \cdot a^{-5}$
c) $b^{-3} \cdot b^5$ **d)** $m^2 \cdot m^{-3}$

4. Write as a single power.
a) $0.5^2 \cdot 0.5^3$ **b)** $0.5^2 \cdot 0.5^{-3}$
c) $\frac{0.5^2}{0.5^3}$ **d)** $\frac{0.5^2}{0.5^{-3}}$

5. Simplify.
a) $\frac{x^4}{x^2}$ **b)** $\frac{x^2}{x^5}$
c) $n^6 \div n^5$ **d)** $\frac{a^2}{a^6}$

6. Simplify.
a) $(n^2)^3$ **b)** $(z^2)^{-3}$
c) $(n^{-4})^{-3}$ **d)** $(c^{-2})^2$

7. Write as a single power.

a) $\left[\left(\frac{3}{5}\right)^3\right]^4$ b) $\left[\left(\frac{3}{5}\right)^3\right]^{-4}$

c) $\left[\left(\frac{3}{5}\right)^{-3}\right]^{-4}$ d) $\left[\left(-\frac{3}{5}\right)^{-3}\right]^{-4}$

8. Simplify.

a) $\left(\frac{a}{b}\right)^2$ b) $\left(\frac{n^2}{m}\right)^3$

c) $\left(\frac{c^2}{d^2}\right)^{-4}$ d) $\left(\frac{2b}{5c}\right)^2$

e) $(ab)^2$ f) $(n^2m)^3$

g) $(c^3d^2)^{-4}$ h) $(xy^{-1})^3$

B

9. Simplify. State the exponent law you used.

a) $x^{-3} \cdot x^4$ b) $a^{-4} \cdot a^{-1}$

c) $b^4 \cdot b^{-3} \cdot b^2$ d) $m^8 \cdot m^{-2} \cdot m^{-6}$

e) $\frac{x^{-5}}{x^2}$ f) $\frac{s^5}{s^{-5}}$

g) $\frac{b^{-8}}{b^{-3}}$ h) $\frac{t^{-4}}{t^{-4}}$

10. Evaluate.

a) $1.5^{\frac{3}{2}} \cdot 1.5^{\frac{1}{2}}$ b) $\left(\frac{3}{4}\right)^{\frac{3}{4}} \cdot \left(\frac{3}{4}\right)^{\frac{5}{4}}$

c) $(-0.6)^{\frac{1}{3}} \cdot (-0.6)^{\frac{5}{3}}$ d) $\left(\frac{4}{5}\right)^{\frac{4}{3}} \cdot \left(\frac{4}{5}\right)^{-\frac{4}{3}}$

e) $\frac{0.6^{\frac{1}{2}}}{0.6^{\frac{3}{2}}}$ f) $\frac{\left(-\frac{3}{8}\right)^{\frac{2}{3}}}{\left(-\frac{3}{8}\right)^{-\frac{1}{3}}}$

g) $\frac{0.49^{\frac{5}{2}}}{0.49^{4}}$ h) $\frac{0.027^{\frac{5}{3}}}{0.027^{\frac{4}{3}}}$

11. Simplify. Explain your reasoning.

a) $(x^{-1}y^{-2})^{-3}$ b) $(2a^{-2}b^2)^{-2}$

c) $(4m^2n^3)^{-3}$ d) $\left(\frac{3}{2}m^{-2}n^{-3}\right)^{-4}$

12. A cone with equal height and radius has volume 1234 cm^3. What is the height of the cone to the nearest tenth of a centimetre?

13. A sphere has volume 375 cubic feet. What is the surface area of the sphere to the nearest square foot?

14. Simplify. Which exponent laws did you use?

a) $\frac{(a^2b^{-1})^{-2}}{(a^{-3}b)^3}$ b) $\left(\frac{(c^{-3}d)^{-1}}{c^2d}\right)^{-2}$

15. Evaluate each expression for $a = -2$ and $b = 1$. Explain your strategy.

a) $(a^3b^2)(a^2b^3)$ b) $(a^{-1}b^{-2})(a^{-2}b^{-3})$

c) $\frac{a^{-4}b^5}{ab^3}$ d) $\left(\frac{a^{-7}b^7}{a^{-9}b^{10}}\right)^{-5}$

16. Simplify.

a) $m^{\frac{2}{3}} \cdot m^{\frac{4}{3}}$ b) $x^{-\frac{3}{2}} \div x^{-\frac{1}{4}}$

c) $\frac{-9a^{-4}b^{\frac{3}{4}}}{3a^2b^{\frac{1}{4}}}$ d) $\left(\frac{-64c^6}{a^9b^{-\frac{1}{2}}}\right)^{\frac{1}{3}}$

17. Identify any errors in each solution for simplifying an expression. Write a correct solution.

a) $(x^2y^{-3})(x^{\frac{1}{2}}y^{-1}) = x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1}$
$= x^1 \cdot y^3$
$= xy^3$

b) $\left(\frac{-5a^2}{b^{\frac{1}{2}}}\right)^{-2} = \frac{10a^{-4}}{b^{-1}}$
$= \frac{10b}{a^4}$

18. Explain how to use a measuring cylinder containing water to calculate the diameter of a marble that fits inside the cylinder.

19. Identify the errors in each simplification. Write the correct solution.

a) $\frac{(m^{-3} . n^2)^{-4}}{(m^2 . n^{-3})^2} = (m^{-5} . n^5)^{-6}$
$= m^{30} . n^{30}$
$= (mn)^{30}$

b) $\left(r^{\frac{1}{2}} . s^{-\frac{3}{2}}\right)^{\frac{1}{2}} . \left(r^{-\frac{1}{4}} . s^{\frac{1}{2}}\right)^{-1} = r^1 . s^{-1} . r^{-\frac{5}{4}} . s^{-\frac{1}{2}}$
$= r^{1-\frac{5}{4}} . s^{-1-\frac{1}{2}}$
$= r^{-\frac{1}{4}} . s^{-\frac{3}{2}}$
$= \frac{1}{r^{\frac{1}{4}} . s^{\frac{3}{2}}}$

20. ISO paper sizes A0, A1, A2, …, are commonly used outside of North America. For any whole number n, the width, in metres, of a piece of An paper is $2^{-\frac{2n+1}{4}}$ and its length, in metres, is $2^{-\frac{2n-1}{4}}$.

a) Write, then simplify expressions to represent the dimensions of each piece of paper. Evaluate each measure to the nearest millimetre.
   i) A3 ii) A4 iii) A5

b) Suppose each piece of paper from part a is folded in half along a line perpendicular to its length. Write, then simplify expressions to represent the dimensions of each folded piece.

c) Compare your results in parts a and b. What do you notice?

C

21. Simplify. Show your work.

a) $\left(\dfrac{a^{-3}b}{c^2}\right)^{-4} \cdot \left(\dfrac{c^5}{a^4b^{-3}}\right)^{-1}$ b) $\dfrac{(2a^{-1}b^4c^{-3})^{-2}}{(4a^2bc^{-4})^2}$

22. If $x = a^{-2}$ and $y = a^{\frac{2}{3}}$, write each expression in terms of a.

a) $\left(x^{\frac{1}{2}}y^{\frac{2}{3}}\right)^2$ b) $\left(x^{\frac{3}{4}} \div y^{-\frac{1}{2}}\right)^3$

23. Write 3 different expressions for each result.

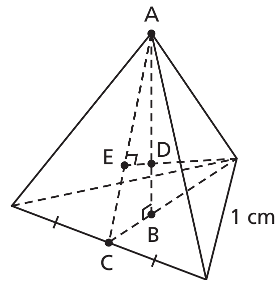
a) $x^{\frac{3}{2}}$ is the product of two powers with rational exponents.

b) $x^{\frac{3}{2}}$ is the quotient of two powers with rational exponents.

c) $x^{\frac{3}{2}}$ is the result of raising a power with a rational exponent to a rational exponent.

24. A regular tetrahedron has edge length 1 cm. It is placed inside a sphere so that all its vertices touch the surface of the sphere. Point D is the centre of the sphere. The measures, in centimetres, of 3 line segments are:

$$AB = \left(\frac{2}{3}\right)^{\frac{1}{2}};\ AC = \frac{3}{2}\left(\frac{1}{3}\right)^{\frac{1}{2}};\ AE = \left(\frac{1}{3}\right)^{\frac{1}{2}}$$



Given that $\triangle$ABC is similar to $\triangle$AED and $\dfrac{AC}{AB} = \dfrac{AD}{AE}$; determine the length of AD.

Reflect

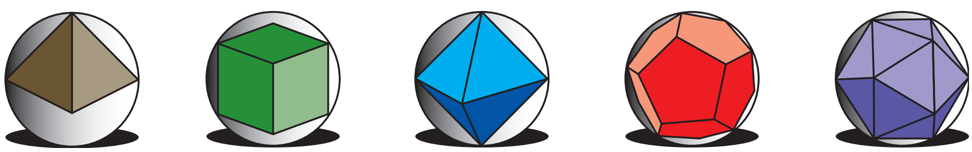
Explain how to apply the exponent laws to simplify algebraic expressions. Use examples to illustrate the types of expressions you can simplify.

THE WORLD OF MATH

Math Fact: Platonic Solids

Platonic solids are the only regular polyhedra that can be placed in a sphere so that each vertex touches the surface of the sphere. In about 300 B.C.E, Euclid used trigonometry, similar triangles, and the Pythagorean Theorem to show the ratio of the edge length of each Platonic solid to the diameter of the sphere:

Tetrahedron Cube Octahedron Dodecahedron Icosahedron



$1 : \left(\frac{3}{2}\right)^{\frac{1}{2}}$ $1 : 3^{\frac{1}{2}}$ $1 : 2^{\frac{1}{2}}$ $1 : \frac{1}{2}\left(3^{\frac{1}{2}} + 15^{\frac{1}{2}}\right)$ $1 : \frac{1}{2}\left(10 + 2(5)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

REVIEW

4.1

1. Evaluate each radical. Why do you not need a calculator?

a) $\sqrt[3]{1000}$ **b)** $\sqrt{0.81}$

c) $\sqrt[6]{64}$ **d)** $\sqrt[4]{\dfrac{81}{625}}$

2. Explain, using examples, the meaning of the index of a radical.

3. Estimate the value of each radical to 1 decimal place. What strategies can you use?

a) $\sqrt{11}$ **b)** $\sqrt[3]{-12}$ **c)** $\sqrt[4]{15}$

4. Identify the number in each case.
a) 5 is a square root of the number.
b) 6 is the cube root of the number.
c) 7 is a fourth root of the number.

5. For $\sqrt[3]{35}$, does its decimal form terminate, repeat, or neither? Support your answer with an explanation.

4.2

6. Tell whether each number is rational or irrational. Justify your answers.

a) -2 **b)** 17 **c)** $\sqrt{16}$
d) $\sqrt{32}$ **e)** 0.756 **f)** $12.\overline{3}$
g) 0 **h)** $\sqrt[3]{81}$ **i)** π

7. Determine the approximate side length of a square with area 23 cm^2. How could you check your answer?

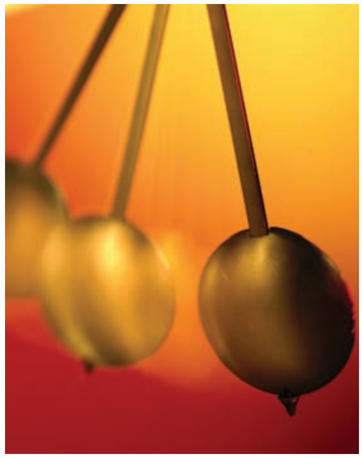
8. Look at this calculator screen.

a) Is the number 3.141 592 654 rational or irrational? Explain.
b) Is the number π rational or irrational? Explain your answer.

9. Place each number on a number line, then order the numbers from least to greatest.

$\sqrt[3]{30}$, $\sqrt{20}$, $\sqrt[4]{18}$, $\sqrt[3]{-30}$, $\sqrt{30}$, $\sqrt[4]{10}$

10. The formula $T = 2\pi\sqrt{\dfrac{L}{9.8}}$ gives the time, T seconds, for one complete swing of a pendulum with length L metres. A clock pendulum is 0.25 m long. What time does the pendulum take to complete one swing? Give the answer to the nearest second.



4.3

11. Write each radical in simplest form.

a) $\sqrt{150}$ **b)** $\sqrt[3]{135}$

c) $\sqrt{112}$ **d)** $\sqrt[4]{162}$

12. Write each mixed radical as an entire radical.

a) $6\sqrt{5}$ **b)** $3\sqrt{14}$

c) $4\sqrt[3]{3}$ **d)** $2\sqrt[4]{2}$

13. Alfalfa cubes are fed to horses to provide protein, minerals, and vitamins.

Two sizes of cubes have volumes 32 cm^3 and 11 cm^3. What is the difference in the edge lengths of the cubes? How can you use radicals to find out?

14. A student simplified $\sqrt{300}$ as shown:

$$\begin{aligned}\sqrt{300} &= \sqrt{3}\cdot\sqrt{100}\\ &= \sqrt{3}\cdot\sqrt{50}\cdot\sqrt{50}\\ &= \sqrt{3}\cdot\sqrt{2}\cdot\sqrt{25}\cdot\sqrt{2}\cdot\sqrt{25}\\ &= 3\cdot 5\cdot\sqrt{2}\cdot 5\\ &= 75\sqrt{2}\end{aligned}$$

Identify the errors the student made, then write a correct solution.

15. Arrange these numbers in order from greatest to least, without using a calculator. Describe your strategy.

$5\sqrt{2}, 4\sqrt{3}, 3\sqrt{6}, 2\sqrt{7}, 6\sqrt{2}$

4.4

16. Show, with examples, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, when n is a natural number and a is a rational number.

17. Express each power as a radical.

a) $12^{\frac{1}{4}}$ **b)** $(-50)^{\frac{5}{3}}$

c) $1.2^{0.5}$ **d)** $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

18. Express each radical as a power.

a) $\sqrt{1.4}$ **b)** $\sqrt[3]{13^2}$

c) $\left(\sqrt[5]{2.5}\right)^4$ **d)** $\left(\sqrt[4]{\frac{2}{5}}\right)^3$

19. Evaluate each power without using a calculator.

a) $16^{0.25}$ **b)** $1.44^{\frac{1}{2}}$

c) $(-8)^{\frac{5}{3}}$ **d)** $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

20. Radioactive isotopes decay. The half-life of an isotope is the time for its mass to decay by $\frac{1}{2}$. For example, polonium-210 has a half-life of 20 weeks. So, a sample of 100 g would decay to 50 g in 20 weeks. The percent, P, of polonium remaining after time t weeks is given by the formula $P = 100(0.5)^{\frac{t}{20}}$. What percent of polonium remains after 30 weeks?

21. Arrange these numbers in order from greatest to least. Describe the strategy you used.

$\sqrt[4]{5}, 5^{\frac{2}{3}}, \sqrt[3]{5}, 5^{\frac{3}{4}}, \left(\sqrt{5}\right)^3$

22. Kleiber's law relates a mammal's metabolic rate while resting, q Calories per day, to its body mass, M kilograms:

$q = 70M^{\frac{3}{4}}$

What is the approximate metabolic rate of each animal?

a) a cow with mass 475 kg

b) a mouse with mass 25 g

4.5

23. a) Identify the patterns in this list.

$81 = 3^4$
$27 = 3^3$
$9 = 3^2$

b) Extend the patterns in part a downward. Write the next 5 rows in the pattern.

c) Explain how this pattern shows that $a^{-n} = \frac{1}{a^n}$ when a is a non-zero rational number and n is a natural number.

24. Evaluate each power without using a calculator.

a) 2^{-2} **b)** $\left(\frac{2}{3}\right)^{-3}$ **c)** $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$

25. Kyle wants to have \$1000 in 3 years. He uses this formula to calculate how much he should invest today in a savings account that pays 3.25% compounded annually: $P = 1000(1.0325)^{-3}$ How much should Kyle invest today?

26. A company designs a container with the shape of a triangular prism to hold 500 mL of juice. The bases of the prism are equilateral triangles with side length s centimetres. The height, h centimetres, of the prism is given by the formula:

$h = 2000(3)^{-\frac{1}{2}}s^{-2}$

What is the height of a container with base side length 8.0 cm? Give your answer to the nearest tenth of a centimetre.

27. When musicians play together, they usually tune their instruments so that the note A above middle C has frequency 440 Hz, called the *concert pitch*. A formula for calculating the frequency, F hertz, of a note n semitones above the concert pitch is:

$$F = 440(\sqrt[12]{2})^n$$

Middle C is 9 semitones below the concert pitch. What is the frequency of middle C? Give your answer to the nearest hertz.

4.6

28. Simplify. Explain your reasoning.

a) $(3m^4n)^2$ b) $\left(\dfrac{x^2y}{y^{-2}}\right)^{-2}$

c) $(16a^2b^6)^{-\frac{1}{2}}$ d) $\left(\dfrac{r^3s^{-1}}{s^{-2}r^{-2}}\right)^{-\frac{2}{3}}$

29. Simplify. Show your work.

a) $(a^3b)(a^{-1}b^4)$ b) $\left(x^{\frac{1}{2}}y\right)\left(x^{\frac{3}{2}}y^{-2}\right)$

c) $\dfrac{a^3}{a^5} \cdot a^{-3}$ d) $\dfrac{x^2y}{x^{\frac{1}{2}}y^{-2}}$

30. Evaluate.

a) $\left(\dfrac{3}{2}\right)^{\frac{3}{2}} \cdot \left(\dfrac{3}{2}\right)^{\frac{1}{2}}$ b) $\dfrac{(-5.5)^{\frac{2}{3}}}{(-5.5)^{-\frac{4}{3}}}$

c) $\left[\left(-\dfrac{12}{5}\right)^{\frac{1}{3}}\right]^6$ d) $\dfrac{0.16^{\frac{3}{4}}}{0.16^{\frac{1}{4}}}$

31. A sphere has volume 1100 cm^3. Explain how to use exponents or radicals to estimate the radius of the sphere.

32. Identify any errors in each solution, then write a correct solution.

a) $$\left(s^{-1}t^{\frac{1}{3}}\right)(s^4t^3) = s^{-1} \cdot s^4 \cdot t^{\frac{1}{3}} \cdot t^3$$
$$= s^{-4}t$$

b) $$\left(\dfrac{4c^{\frac{1}{3}}}{d^3}\right)^{-3} = \dfrac{-12c^{-1}}{d^0}$$
$$= -12c^{-1}$$
$$= \dfrac{1}{12c}$$

THE WORLD OF MATH

Historical Moment: The Golden Ratio

The ratio, $\dfrac{1+\sqrt{5}}{2}$: 1, is called the *golden ratio*.

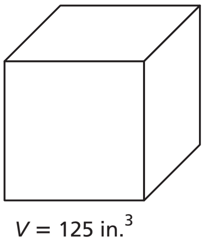
Buildings and pictures with dimensions in this ratio are often considered visually pleasing and "natural." The Greek sculptor Phidias used the golden ratio for the dimensions of his sculptures. His 42-ft. high statue of the Greek god Zeus in the temple in Olympia, created in about 435 B.C.E., was one of the Seven Wonders of the Ancient World. The number $\dfrac{1+\sqrt{5}}{2}$ is often called "phi" after the first Greek letter in "Phidias."

PRACTICE TEST

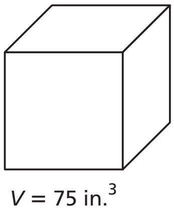
For questions 1 and 2, choose the correct answer: A, B, C, or D

1. The volume V cubic inches of each cube is given. For which cube is the edge length an irrational number?

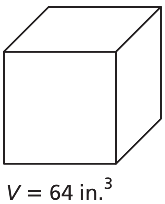
A.



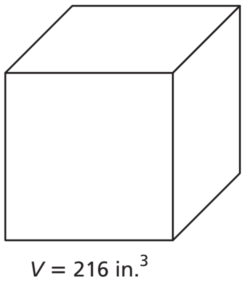
B.



C.



D.



2. Which number is rational?

A. $\sqrt{0.09}$ **B.** $\sqrt{50}$ **C.** $\sqrt[3]{-\frac{64}{121}}$ **D.** π

3. **a)** Which is greater, $\sqrt{70}$ or $5\sqrt{3}$? Justify your answer.
b) Sketch a number line to illustrate the numbers in part a.

4. Evaluate without using a calculator.

a) $\sqrt[4]{\frac{256}{81}}$ **b)** $(-4)^{-2}$ **c)** $0.81^{\frac{3}{2}}$ **d)** $16^{-\frac{1}{2}}$

5. Write $44^{\frac{1}{2}}$ as a radical in simplest form.

6. A student simplified $\frac{x^{-1}y^3}{xy^{-2}}$ as follows:

$$\begin{aligned}\frac{x^{-1}y^3}{xy^{-2}} &= x^{-1+1} \cdot y^{3-2} \\ &= x^0y^1 \\ &= y\end{aligned}$$

Is the student correct? If not, describe any errors and write a correct solution.

7. Simplify each expression. Write your answers using positive exponents.

a) $(p^{-2}q^{-1})^2\left(pq^{\frac{1}{2}}\right)^2$ **b)** $\left(\frac{c^6d^5}{c^3d^4}\right)^{-\frac{1}{3}}$

8. Scientists use the formula $d = 0.099m^{\frac{9}{10}}$ to calculate the volume of water, d litres, that a mammal with mass m kilograms should drink in 1 day. Calculate how much water a 550-kg moose should drink in one day.