

**Arithmetic Sequences**

A **sequence** is an ordered list of objects that follow a pattern or rule.

Example: Find the pattern for each sequence below.

a) 4, 7, 10, 13      *add 3*

*(Arrows show +3 between 4 and 7, 7 and 10, 10 and 13)*

b) 5, -1, -7, -13, ...      *subtract 6*

*(Arrows show -6 between 5 and -1, -1 and -7, -7 and -13)*

c) 2, 4, 8, 16, ...      *multiply by 2*

*(Arrows show x2 between 2 and 4, 4 and 8, 8 and 16)*

Example: Determine the first 4 terms of an **arithmetic sequence** if the first term is 5 and the common difference is  $\frac{1}{2}$ .

*5, 5½, 6, 6½*

*(Arrows show +½ between 5 and 5½, 5½ and 6, 6 and 6½)*

**General Term for an Arithmetic Sequence**

$t_n = a + (n - 1)d$

a = the first term

d = the common difference

n = the number or terms

$t_n$  = the general term or the  $n^{\text{th}}$  term

Example: Determine the general term for the following arithmetic sequences.

a) 7, 14, 21, ...      *a = 7*  
*d = 14 - 7 = 7*

b) 8, 3, -2, ...      *a = 8*  
*d = 3 - 8 = -5*

$t_n = 7 + (n - 1)7$   
 $= 7n$

$t_n = 8 + (n - 1)(-5)$   
 $= 13 - 5n$

Example: For the sequence defined by  $t_n = -5n + 13$ , determine

a)  $t_1 = a = -5(1) + 13$   
 $= 8$

b)  $t_5 = -5(5) + 13$   
 $= -12$

c)  $t_{12} = -5(12) + 13$   
 $= -47$

Example: Determine  $a$  and  $d$  for the following arithmetic sequence and then find the missing terms.

*\* Show proper algebraic work \**

\_, 2, \_, \_, 17  
 $\uparrow \quad \uparrow \quad \uparrow$   
 $+d \quad +d \quad +d$

$2 + 3d = 17$   
 $3d = 15$   
 $d = 5$

and  $a + d = 2$   
 $a + 5 = 2$   
 $a = -3$

missing terms: 7, 12

Example: Two terms of an arithmetic sequence,  $t_4 = 24$  and  $t_{10} = 66$  are given. Find  $t_1$ .

$24 + 6d = 66$   
 $6d = 42$   
 $d = 7$

$t_4 = 24$   
 $a + (3-1)(7) = 24$   
 $a + 14 = 24$

$a = 10 = t_1$

Example: For the arithmetic sequence: 3, 8, 13, 18, ...; which term has a value of 123?

$n = ?$   
 $a = 3$   
 $d = 5$   
 $123 = 3 + (n-1)5$   
 $120 = 5(n-1)$   
 $24 = n-1$   
 $n = 25$

Example: You started a sit-up routine that followed an arithmetic sequence. On the 6<sup>th</sup> day of the program, you performed 11 sit-ups. On the 15<sup>th</sup> day, you did 29 sit-ups.

- a) Write a general term that relates the number of sit-ups to the number of days.
- b) If your goal is to be able to do 100 sit-ups, on which day will you accomplish this?
- c) What assumptions did you make to answer part b)?

$t_6 = 11, t_{15} = 29$   
 $11 + 9d = 29$   
 $9d = 18$   
 $d = 2$

$t_6 = 11$   
 $a + (6-1)2 = 11$   
 $a = 1$

a)  $t_n = 1 + (n-1)2$   
 $= 2n - 1$   
 b)  $100 = 2n - 1$   
 $99 = 2n$

$n = 49.5$  day 50