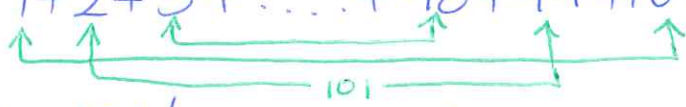


Arithmetic Series (Part One)

What is the sum of the numbers from 1 to 100?

Gauss did this: $1+2+3+\dots+98+99+100$

All pairs add to 101! $50(101) = 5050$

A **series** is the sum of the terms in a sequence. S_n denotes the sum of the first n terms.

In general, the sum of the first n terms of an arithmetic series is

$$S_n = \frac{n}{2}(a + t_n) \quad \text{where } a = \text{first term, } t_n = \text{last term, and } n = \text{the number of terms}$$

Also, because $t_n = a + (n-1)d$, then

$$S_n = \frac{n}{2}(2a + (n-1)d) \quad \text{where } a = \text{first term, } n = \text{the number of terms, and} \\ d = \text{common difference.}$$

Example: Determine the sum of the each arithmetic series.

a) $7+14+21+28+35+42$ $a=7, n=6$
 $d=7$

$$S_6 = \frac{6}{2}(7+42) \\ = 147$$

b) $42+37+32+\dots$ Find S_{18} .

$$t_{18} = 42 + (18-1)(-5) \\ = -43$$

c) $-13-10-7-\dots+62$ How many terms are there?
 $n=?$

$$t_n = (-13) + (n-1)(3) = 62 \\ 3(n-1) = 75 \\ n-1 = 25 \\ n = 26$$

$$S_{26} = \frac{26}{2}(-13+62) = 637$$

Example: How many terms are there in the arithmetic series $4 + \dots + 10$, if the sum is 49?

$$\begin{aligned} a &= 4 & S_n &= \frac{n}{2}(a + t_n) \\ t_n &= 10 & 49 &= \frac{n}{2}(4 + 10) \\ S_n &= 49 & \frac{n}{2} &= 3.5 \\ n &= ? & n &= 7 \end{aligned}$$

Example: Determine the value of the first term, a , of the arithmetic series that has $d = -3$ and $S_{18} = 279$.

$$\begin{aligned} S_n &= \frac{n}{2}(2a + (n-1)d) \\ 279 &= \frac{18}{2}(2a + (18-1)(-3)) \\ 31 &= 2a - 51 \\ 2a &= 82 \\ a &= 41 \end{aligned}$$

Example: Determine the common difference, d , if $a = 4$ and $S_{15} = 322.5$.

$$\begin{aligned} S_n &= \frac{n}{2}(2a + (n-1)d) \\ 322.5 &= \frac{15}{2}(2(4) + (15-1)d) \\ 43 &= (8 + 14d) \\ d &= 2.5 \end{aligned}$$

Example: Ryan's grandparents loaned him the money to purchase a bike. He agreed to repay \$25 at the end of the first month, \$30 at the end of the second month, \$35 at the end of the third month, and so on. Ryan repaid the loan in 12 months. How much did the bike cost?

$$\begin{aligned} 25 + 30 + 35 + \dots + t_{12} & \quad S_{12} = ? \quad \begin{matrix} a = 25 \\ d = 5 \end{matrix} \\ S_n &= \frac{n}{2}(2a + (n-1)d) \\ S_{12} &= \frac{12}{2}(2(25) + (12-1)5) \\ &= \$630 \end{aligned}$$