Foundations & Pre-Calculus Math 10
Sequences and Series: Lesson 4

Geometric Sequences

A **geometric sequence** is a list of numbers in which the ratio of consecutive terms is constant (called the *common ratio*).

Example: Find the common ratio for the following geometric sequences.

a) 1, 3, 9, 27, ...

$\frac{3}{1} = 3$

b) 2, -10, 50, -250, ...

$\frac{-10}{2} = -5$

c) 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, ...

$\frac{3}{6} = \frac{1}{2}$

The *general term* of a **geometric sequence** is

$t_n = ar^{n-1}$ where a = first term, n = number of terms, r = common ratio, and t_n = the nth term or the general term

Example: Determine the general term and the 12th term for the following geometric sequences.

a) 1, 3, 9, 27, ...

$a=1, r=3, n=12$

$t_n = 1 \cdot 3^{n-1}$
$= 3^{n-1}$

$t_{12} = 3^{12-1}$
$= 3^{11}$
$= 177\,147$

b) 2, -10, 50, -250, ...

$a=2, r=-5, n=12$

$t_n = 2(-5)^{n-1}$

$t_{12} = 2(-5)^{12-1}$
$= -97\,656\,250$

c) 6, 3, $\frac{3}{2}$, $\frac{3}{4}$, ...

$a=6, r=\frac{1}{2}, n=12$

$t_n = 6\left(\frac{1}{2}\right)^{n-1}$

$t_{12} = 6\left(\frac{1}{2}\right)^{12-1}$
$= \frac{6}{2^{11}} = \frac{3 \cdot 2}{2^{11}} = \frac{3}{2^{10}}$
$= \frac{3}{1024}$

Example: Determine the first 4 terms for each geometric sequence.

a) $t_n = 4(-3)^{n-1}$ (a, r)

4, -12, 36, -108

b) $t_n = 100\left(\frac{1}{2}\right)^{n-1}$ (a, r)

100, 50, 25, 12.5

Example: In a geometric sequence, $a = 5$ and $t_5 = 1280$.

a) Determine t_2 and t_6.

$t_5 = ar^{5-1} = 1280$

$r^4 = 256$

$r = \sqrt[4]{256}$

$r = \pm 4$

$t_2 = 20 \quad t_6 = 5(\pm 4)^5 = \pm 5120$

b) If the last term is 20 480, how many terms are in the geometric sequence?

$t_n = 20480, n = ?$

$5(4)^{n-1} = 20480$

$4^{n-1} = 4096$ ← requires logarithms

For now: $4^6 = 4096$

$n = 7$

Example: Insert 3 numbers between 8 and 128, so the five numbers form a geometric sequence.

8, ___, ___, ___, 128

×r ×r ×r ×r

$8(r)^4 = 128$

$r^4 = 16$

$r = \pm 2$

16, 32, 64 or −16, 32, −64

Example: Suppose a person is given 1¢ on the first day of April; 3¢ on the second day; 9¢ on the third day, and so on. This pattern continues throughout April. About how much money will the person be given on the 20th day of April?

1, 3, 9, ...

$a = 1$

$r = 3$

$t_{20} = 1 \cdot 3^{19}$

$= 1\,162\,261\,467$ ¢

Example: A ball is dropped from a height of 3.0 m. After each bounce it rises to 75% of its previous height.

→ 1st up

a) Write the first term and the common ratio of the geometric sequence.

$a = 3(0.75) = 2.25 \quad r = 0.75$

b) Write the general term for the sequence in part a).

$t_n = 2.25(0.75)^{n-1}$

c) What height does the ball reach after the 6th bounce?

$t_6 = 2.25(0.75)^{6-1}$

$= 0.53\text{m}$