Foundations & Pre-Calculus 10
Sequences and Series: Lesson 5

Geometric Series

A **geometric series** is the sum of the terms of a geometric sequence.

The *sum* of a geometric series is

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

where a = first term, n = number of terms, r = common ratio, and S_n = the sum of the first n terms

Example: Determine the sum of the first 8 terms for the following geometric series.

a) $1 + 3 + 9 + 27 + \ldots$

$a=1, r=3, n=8$

$S_8 = \frac{1(3^8-1)}{3-1}$

$= 3280$

b) $2 - 10 + 50 - 250 \ldots$

$a=2, r=(-5), n=8$

$S_8 = \frac{2((-5)^8-1)}{-5-6}$

$= \frac{781248}{11}$

OR

$\doteq -71022.545$

c) $6 + 3 + \frac{3}{2} + \frac{3}{4} + \ldots$

$a=6, r=\frac{1}{2}, n=8$

$S_8 = \frac{6((\frac{1}{2})^8-1)}{\frac{1}{2}-1}$

$= 11.953125$

Example: The sum of the first 10 terms of a geometric series is -29524. The common ratio is -3. Determine the first term.

$S_{10} = -29524, n = 10$

$r = -3$

$a = ?$

$\frac{a((-3)^{10}-1)}{-3-1} = -29524$

$a(59048) = 118096$

$a = 2$

Example: Determine the sum of this geometric series: 6 + 12 + 24 + ...+ 12 288

$a=6$ $S_n=?$ $t_n = ar^{n-1}$

$r=2$

$n=?$

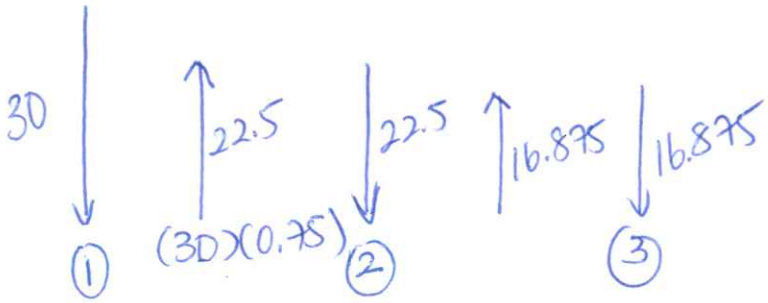
$6(2)^{n-1} = 12\,288$

$2^{n-1} = 2048$ ← requires logarithms

$2^{11} = 2048 \rightarrow n = 12$

$$S_{12} = \frac{6(2^{12}-1)}{2-1} = 24\,570$$

Example: A basketball dropped from a height of 30m bounces to 75% of its previous height on each bounce. The total vertical distance travelled is made up of upward bounces and downward drops. Draw a diagram to represent this situation. What is the total vertical distance the ball has travelled when it hits the floor for the 6th time? Answer to the nearest tenth of a metre.



$a=30$

$r=0.75$

$n=6$

terms are the "down" distance

$$S_6 = \frac{30(0.75^6-1)}{0.75-1}$$

$= 98.642\,578\,13$

↑ this is only down, include "up"

← only drops initial 30m once!

Total vertical: $2S_6 - 30 \doteq 167\text{m}$