

Arithmetic Sequences and Series Review

$$t_n = a + (n-1)d \text{ and } S_n = \frac{n}{2}(a + t_n)$$

1) Determine which of the following sequences are arithmetic. If the sequence is arithmetic, determine the common difference and the next 4 terms.

a) 8, 11, 14, ...

$$d=3$$

17, 20, 23

b) 2, 6, 18, ...

No

c) 8, 4, 2, ...

No

d) -2.5, -1.1, 0.3, ...

$$d=1.4$$

1.7, 3.1, 4.5

e) 20, 16, 12, ...

$$d=-4$$

8, 4, 0

f) 1, 1, 2, 3, 5, ...

No

2) Given the sequence 10, 7, 4, ...

$$a=10, d=-3$$

a) determine the 21st term.

$$t_{21} = 10 + (21-1)(-3) = -50$$

b) which term is -77? $n=?$

$$\begin{aligned} -77 &= 10 + (n-1)(-3) \\ -87 &= -3(n-1) \\ 29 &= n-1 \\ n &= 30 \end{aligned}$$

3) Find the missing terms in each sequence.

a) $\underline{4}$, 10, 16, $\underline{22}$, $\underline{28}$

$$\begin{aligned} 10+d &= 16 \\ d &= 6 \end{aligned}$$

b) $\underline{13}$, 9, $\underline{5}$, $\underline{1}$, -3

$$\begin{aligned} 9+3d &= -3 \\ 3d &= -12 \\ d &= -4 \end{aligned}$$

c) 5, $\underline{18}$, $\underline{31}$, 44

$$\begin{aligned} 5+3d &= 44 \\ 3d &= 39 \\ d &= 13 \end{aligned}$$

d) -3, $\underline{0.75}$, $\underline{4.5}$, $\underline{8.25}$, 12

$$\begin{aligned} -3+4d &= 12 \\ 4d &= 15 \\ d &= 3.75 \end{aligned}$$

e) 3.4, $\underline{1.6}$, $\underline{-0.2}$, -2, $\underline{-3.8}$

$$\begin{aligned} 3.4+3d &= -2 \\ 3d &= -5.4 \\ d &= -1.8 \end{aligned}$$

f) 1.5, $\underline{1}$, $\underline{0.5}$, 0, $\underline{-0.5}$

$$\begin{aligned} 1.5+3d &= 0 \\ 3d &= -1.5 \\ d &= -0.5 \end{aligned}$$

4) An arithmetic sequence has a 10th term of 18 and a 14th term of 30. Find the common difference and the first term.

$$t_{10} = 18$$

$$18+4d=30$$

$$18 = a + (10-1)d$$

$$t_{14} = 30$$

$$4d=12$$

$$18 = a + 3(9)$$

$$d=3$$

$$18 = a + 27$$

$$a = -9$$

5) Find the sum of the following series.

a) 5, 9, 13, ..., 101 $a=5, d=4$

$$101 = 5 + (n-1) \cdot 4 \quad S_{25} = \frac{25(5+101)}{2}$$

$$24 = 4(n-1) \quad = 1325$$

$$n=25$$

b) 83, 80, 77, ..., 5 $a=83, d=-3$

$$5 = 83 + (n-1)(-3) \quad S_{27} = \frac{27(83+5)}{2}$$

$$-78 = -3(n-1) \quad = 1188$$

$$26 = n-1$$

$$n=27$$

6) Find the sum of the following.

a) The first 16 terms of 4, 11, 18 ... $a=4, d=7$

$$t_{16} = 4 + (16-1)7 = 109$$

$$S_{16} = \frac{16(4+109)}{2} = 904$$

b) The first 10 terms of 19, 13, 7, ... $a=19, d=-6$

$$t_{10} = 19 + (10-1)(-6) = -35$$

$$S_{10} = \frac{10}{2}(19 + (-35)) = -80$$

7) Find the sum of the first 100 odd numbers.

$$1, 3, 5, \dots$$

$$a=1, d=2$$

$$t_{100} = 1 + (100-1)2 = 199$$

$$S_{100} = \frac{100}{2}(1 + 199)$$

$$= 10000$$

8) Jerry deposited \$20,000 on an investment that will give \$1,750 for every year that his money stays in the account. How much money will he have in his account by the end of year 8?

$$1750, 3500, \dots$$

$$a=1750$$

$$d=1750$$

$$t_8 = 1750 + (8-1)(1750) = 14000$$

$$S_8 = \frac{8}{2}(1750 + 14000) = 63000$$

$$20000 + 63000 = \$83000$$

9) In his piggy bank, Bingo dropped \$1.00 on May 1, \$1.75 on May 2, \$2.50 on May 3 and so on until the last day of May. $a=1, d=0.75$

a) How much did he drop in his piggy bank on May 19?
 $t_{19} = 1 + (19-1)(0.75) = \14.50

b) What was his total deposit in his piggy bank for the month of May?

$$t_{31} = 1 + (31-1)(0.75) = 23.50$$

$$S_{31} = \frac{31}{2}(1 + 23.50) = \$379.75$$