The Sum of an Arithmetic Series - Part 2

By substituting the formula $t_n = a + (n-1)d$ into the formula $S_n = \frac{n}{2}(a + t_n)$, we can generate a second formula for S_n:

$$S_n = \frac{n}{2}(a + \underbrace{a + (n-1)d}_{t_n})$$

$$\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$$

Note: We can expand further, but it would get too messy

This formula can be used to solve problems involving linear systems (Chapter 7).

Example: The sum of the first 2 terms of an arithmetic series is 6 and the sum of the first 5 terms is 45. Determine the first 5 terms of the series.

$S_2 = 6$ and $S_5 = 45$
$n = 2$ $n = 5$

$\frac{2}{2}(2a + (2-1)d) = 6$

$2a + d = 6$

$\frac{5}{2}(2a + (5-1)d) = 45$

$\frac{5}{2}(2a + 4d) = 45$

$5a + 10d = 45$

A system of linear equations.

Choose substitution:

$d = 6 - 2a$

$5a + 10(6 - 2a) = 45$
$5a + 60 - 20a = 45$
$60 - 15a = 45$
$-15a = -15$
$a = 1$

$d = 6 - 2(1)$
$d = 4$

First 5 terms

1, 5, 9, 13, 17

Example: The sum of the first 9 terms of an arithmetic series is 216 and the 4th term is 20. Determine the first 3 terms.

$S_9 = 216$ $\quad t_4 = 20$

$n = 9$

$\frac{9}{2}(2a + (9-1)d) = 216$

$\frac{9}{2}(2a + 8d) = 216$

$9a + 36d = 216$

$a + (4-1)d = 20$

$a + 3d = 20$

$a = 20 - 3d$

$9(20-3d) + 36d = 216$

$180 - 27d + 36d = 216$

$180 + 9d = 216$

$9d = 36$

$d = 4$

$a = 20 - 3(4)$

$a = 8$

First 3 terms: 8, 12, 16